5.1.21. The velocity of a particle moving along the $x$-axis is given by $f(t) = 6 - 2t$ cm/sec. Use a graph of $f(t)$ to find the exact change in position of the particle from time $t = 0$ to $t = 4$ seconds.

**Solution.** As usual we take right to be positive on the $x$-axis and left to be negative. The graph of the velocity is a straight line $y = 6 - 2t$:

![Plot of velocity versus time]

The exact change of position from time $t = 0$ to time $t = 4$ is the area under the curve $y = f(t)$ above the $x$-axis minus the area above the curve $y = f(t)$ and below the $x$-axis. This consists of two triangles: the first triangle has base three and height six, whereas the second has base one and height two. So the area of the first triangle is nine, whereas the second has area one. So the change in position is $9 - 1 = 8$ cm to the right.

5.1.22. A baseball thrown directly upward at 96 ft/sec has velocity $v(t) = 96 - 32t$ ft/sec at time $t$ seconds.

(a) Graph the velocity from time $t = 0$ to time $t = 6$.
(b) When does the baseball reach the peak of its flight?
(c) How high is the baseball at time $t = 5$?

**Solution.** (a) The graph of the velocity is shown below:
(b) The baseball reaches the peak of its flight when the velocity is zero, and this happens at time $t = 3$ seconds, since $96 - 32t = 0$ when $t = 3$ seconds. At that time, the height is equal to the area under the curve $v(t)$ and above the $x$-axis for $0 \leq t \leq 3$. From the graph above, this is the area of the triangle with base three and height 96, and so the answer is $\frac{1}{2}(3 \cdot 96) = 144$ ft.

(c) At time $t = 5$, the height of the baseball (which is the same as the change in position, or distance travelled) is the area of the triangle we used in (b) minus the area of the triangle under the curve $y = v(t)$ for $3 \leq t \leq 5$, since this part of the curve is under the $t$-axis. This triangle has area $\frac{1}{2}(2 \cdot 54) = 54$, and therefore the height is $144 - 54 = 90$ feet.

5.2.1. Using Figure 5.2.4, draw rectangles representing each of the following Riemann sums for the function $f$ on the interval $0 \leq t \leq 8$. Calculate the value of each sum.

(a) The left hand sum with $\Delta t = 4$ is $32 \cdot 4 + 24 \cdot 4 = 224$.
(b) The right hand sum with $\Delta t = 4$ is $24 \cdot 4 + 0 \cdot 4 = 96$.
(c) The left hand sum with $\Delta t = 2$ is $32 \cdot 2 + 30 \cdot 2 + 24 \cdot 2 + 14 \cdot 2 = 248$.
(d) The left hand sum with $\Delta t = 2$ is $30 \cdot 2 + 24 \cdot 2 + 14 \cdot 2 + 0 \cdot 2 = 184$.

5.2.11. Use a calculator or a computer to find the value of the definite integral to four decimal places.

$$\int_{-1}^{1} e^{-x^2} \, dx$$

Using a calculator we get $1.493648266 \approx 1.49$. 2