In order for a matrix $B$ to be the inverse of $A$, both equations $AB = I$ and $BA = I$ must be true. TRUE We’ll see later that for square matrices $AB=I$ then there is some $C$ such that $BC=I$. CHALLENGE: Can you find an inverse for any non-square matrix. If so find one, if not explain why. Also in above statement about square matrices, does $C=A$?

If $A$ and $B$ are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of $AB$. FALSE $AB^{-1} = B^{-1}A^{-1}$. Remember ”shoes and socks.”

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ab – cd \neq 0$, then $A$ is invertible. False. $A$ is invertible is $ad – bc \neq 0$

If $A$ is an invertible $n \times n$ matrix, then the equation $Ax = b$ is consistent for each $b$ in $\mathbb{R}^n$. TRUE. Since $A$ is invertible we have that $x = A^{-1}b$
Each elementary matrix is invertible. True. Let $K$ be the elementary row operation required to change the elementary matrix back into the identity. If we preform $K$ on the identity, we get the inverse.
A product of invertible \( n \times n \) matrices is invertible, and the inverse of the product of their matrices in the same order. FALSE. It is invertible, but the inverses in the product of the inverses in the reverse order.

If \( A \) is invertible, then the inverse of \( A^{-1} \) is \( A \) itself. TRUE

If \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) and \( ad = bc \), then \( A \) is not invertible. TRUE. \( A \) is invertible is \( ad - bc \neq 0 \) but if \( ad = nc \) then \( ad - bc = 0 \) so \( A \) is not invertible.

If \( A \) can be row reduced to the identity matrix, then \( A \) must be invertible. TRUE. The algorithm presented in this chapter tells us how to find the inverse in this case.

If \( A \) is invertible, then elementary row operations then reduce \( A \) to to the identity also reduce \( A^{-1} \) to the identity. FALSE. They also reduce the identity to \( A^{-1} \).
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- If the equation $Ax = 0$ has only the trivial solution, then $A$ is row equivalent to the $n \times n$ identity matrix. TRUE From Thm 8

- If the columns of $A$ span $\mathbb{R}^n$, then the columns are linearly independent. TRUE Again from Thm 8. Also is $n$ vector span $\mathbb{R}^n$ they must be linearly independent.

- If $A$ is an $n \times n$ matrix then the equation $Ax = b$ has at least one solution for each $b$ in $\mathbb{R}^n$. FALSE we need to know more about $A$ like if it in invertible (or anything else in Thm 8)

- If the equation $Ax = 0$ has a nontrivial solution, then $A$ has fewer than $n$ pivot positions. TRUE This comes from the "all false" part of THM 8. (The statements are either all true or all false.)

- If $A^T$ is not invertible, then $A$ is not invertible. TRUE Also from the all false part of theorem 8.
If there is an $n \times n$ matrix $D$ such that $AD = I$, then there is also an $n \times n$ matrix $C$ such that $CA = I$. TRUE Thm 8

If the columns of $A$ are linearly independent, then the columns of $A$ span $\mathbb{R}^n$. TRUE Thm 8

If the equation $Ax = b$ has at least solution for each $b$ in $\mathbb{R}^n$, then the solution is unique for each $b$. TRUE Thm 8

If the linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$ then $A$ has $n$ pivot points. FALSE. Since $A$ is $n \times n$ the linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$. This doesn’t tell us anything about $A$.

If there is a $b$ in $\mathbb{R}^n$ such that the equation $Ax = b$ is inconsistent, then the transformation $x \mapsto Ax$ is not one-to-one. TRUE Thm 8
An $n \times n$ determinant is defined by determinants of $(n-1) \times (n-1)$ submatrices. TRUEish I am a little unhappy about the defined by term in here since they are not completely defined by these submatrices.

The $(i,j)$-cofactor of a matrix $A$ is the matrix $A_{ij}$ obtained by deleting from $A$ its $i$th row and $j$th column. FALSE The cofactor is the determinant of this $A_{ij}$ times $-1^{i+j}$.

The cofactor expansion of $\det A$ down a column is the negative of the cofactor expansion along a row. FALSE We can expand down any row or column and get same determinant.

The determinant of a triangular matrix is the sum of the entries of the main diagonal. FALSE It is the product of the diagonal entries.