Math 20C
Midterm Examination 1
February 3, 2012

Turn off and put away your cell phone.
No calculators or any other devices are allowed on this exam.
You may use one 8.5 \times 11 page of handwritten notes, but no other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

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1. (9 points) Suppose that $\mathbf{u} = <2, 0, -1>$, $\mathbf{v} = <2, -1, 2>$, $\mathbf{w} = <3, 1, 0>$.
Compute

(a) (2 points) $\mathbf{u} \cdot \mathbf{v}$

(b) (3 points) $\mathbf{v} \times (\mathbf{w} + 2\mathbf{v})$

(c) (2 points) $||\mathbf{v}||$

(d) (2 points) $\mathbf{e}_v$, the unit vector in the direction of $\mathbf{v}$
2. (10 points)
For each of the following questions, circle T if the statement is true, and F if the statement is false. You do not need to justify your answers for this question. You will earn:

- 2 points for a correct answer
- 1 point for any question left blank
- 0 points for an incorrect answer

For all questions, \( \mathbf{u}, \mathbf{v}, \text{ and } \mathbf{w} \) are vectors in \( \mathbb{R}^3 \).

T F If \( \mathbf{u} \cdot \mathbf{v} < 0 \), then the angle between \( \mathbf{u} \) and \( \mathbf{v} \) is obtuse.

T F There is exactly 1 plane containing the points \((0, 1, 0), (1, 0, 0), \text{ and } (0, 0, 1)\).

T F There is exactly 1 plane containing the points \((1, 1, 1), (3, 3, 3), \text{ and } (\pi, \pi, \pi)\).

T F If \( \mathbf{v} \perp \mathbf{w} \), then \( \mathbf{v} \times \mathbf{w} = \vec{0} \)

T F If \( \mathbf{u} \perp \mathbf{v} \), and \( \mathbf{v} \perp \mathbf{w} \), then \( \mathbf{u} \perp \mathbf{w} \)

T F If \( f(1) = \pi, f'(1) = 2, \mathbf{r}'(2) = < 3, 1, 5 > \) and \( \mathbf{r}'(\pi) = < 3\pi, \pi, \pi^2 + 1 > \), then \( \frac{d}{dt} [\mathbf{r}(f(t))]_{t=1} \), the derivative evaluated at \( t = 1 \), is \( < 6, 2, 10 > \).
3. (5 points)

Find a vector parametrization of the line tangent to the curve

\[ \mathbf{r}(t) = (5t, t^2 - t + 1, 2t^3) \]

at \( t = 1 \).
4. (16 points)
A peregrine falcon is circling, looking for prey. The position of the falcon at time $t$ is given by the vector-valued function
\[ r(t) = (3 \sin(2t), 3 \cos(2t), 4) \]

(a) (4 points) Compute the velocity at time $t$.

(b) (4 points) Compute the acceleration at time $t$.

(c) (4 points) Compute the distance the falcon travels between $t = 0$ and $t = \pi$.

(d) (4 points) At time $t = \pi$, is the falcon speeding up, slowing down, or staying constant speed? Justify your answer.
5. (10 points) Consider the plane $P$ given by the equation $2(x - 1) + 3(y + 2) - (z - 7) = 0$ and the line $l(t) = p_0 + tv$.

(a) (2 points) Find a value of $v$ that makes the line perpendicular to the plane $P$.

(b) (4 points) Find a value of $v$ that makes the line parallel to the plane $P$.

(c) (4 points) Find values for $v$ and $p_0$ that make the line lie in the plane $P$. 