Is it that burst of discovery, the long haze of confusion, picking at a problem, and then the flash of understanding, of knowing? Or is it a certain beauty in the elegance of argument, ideas like the arc of a white sail over azure waters? It is hard to say what, exactly, makes mathematics so appealing and so fun, but I hope that my students will, in addition to gaining familiarity with important ideas and techniques, at least understand that mathematics can be fun.

While I am sympathetic to some criticisms of lectures, I do think they can be a powerful tool, and they are a good way for an instructor to convey their own wonder and enthusiasm to the students, who sometimes lack the perspective to recognize an exciting or novel idea. For instance, I was pleased that my multivariable calculus students actually found the non-commutativity of the cross product remarkable when I emphasized it (and they remembered it).

I am not overly fond of approaching math from an axiomatic point of view – I find it much more natural and intuitive to work through examples and gather the overarching ideas from there, so this is how I try to lecture. Frequently, I begin with a topic previously covered, and talk the students through why we might think of extending it, or generalizing it. In UCSD’s multivariable calculus course, Math 20C, virtually every topic covered has an analog from one-variable calculus, so this was very natural. I then state any relevant theorems and definitions, and give some examples to illustrate their application. If it is helpful and appropriate, I will sketch proofs. Spelling out every detail of a proof in a lecture is not especially helpful, particularly for lower-division courses; the listener has to struggle with the proof nearly as much as if they are reading the proof, only without the needed time. But giving the idea can help with understanding, and if the listener wants to read the proof, they will be armed with an outline of how it goes.

Wrestling with problems is really an individual affair, so perhaps to some degree, all an instructor can do is assign some challenging homework problems (and perhaps a few exam questions) and hope for the best. But I am excited about power of small groups as a venue for this struggle and all the learning that comes with it, and I would like to move my teaching style more in this direction. I think one of my best quarters as a TA was when I relied heavily on groups of two or three (small enough that everyone has to participate). The students developed a great rapport with each other and with me, and best of all, were comfortable coming to the board to present their own solutions. My difficulty was that students, at the end of the quarter, said they thought the groups were not a good use of time because it was something they could do outside of class. So I reverted to a more conventional style of taking questions in section and working them on the board myself, but I am still interested in thinking of a way to incorporate groups in a way that students think is worthwhile. Perhaps eventually, I would even like to try a modified Moore method, in which students learn material ordinarily covered in lectures by going through worksheets in small groups.

I do not believe there is a perfect recipe for successful teaching, and like any interested practitioner, I am constantly tinkering with my instruction. I find many ideas worth pursuing further, from brain-based learning to computer-based systems like the Khan academy. But no matter what techniques I use, I hope my students will learn not just basic skills, but will share some of my puzzler’s delight, and wonder at the sheer beauty and strangeness of mathematics.