1. (40 pts.) Let \( R \) be the region between the two parabolas \( y = x^2 \) and \( x = y^2 \).
Let \( V \) be the volume obtained when \( R \) is rotated about the \( y \)-axis.

(a) Sketch the region \( R \). Include in your drawing the coordinates of the point where the parabolas intersect.

Set up (but do not evaluate) integrals for the following.

(b) The arc length of the boundary of \( R \) that is on the parabola \( y = x^2 \).

(c) The volume of \( V \).

(d) The surface area of \( V \). Be careful: \( V \) has what might be called an inner and outer surface. The surface area is the sum of the areas of these two surfaces.
2. (20 pts.) Given the two curves \( r = 2 \) and \( r = 4 \cos \theta \) in polar coordinates.

(a) Find the polar coordinates of the points where the curves intersect.

(b) Set up (but do not evaluate) an integral for the area that lies inside the curve \( r = 4 \cos \theta \) but outside the curve \( r = 2 \); that is, the area of the region for which \( 2 \leq r \leq 4 \cos \theta \).

3. (30 pts.) Express each of the following in the form \( a + bi \) with \( a \) and \( b \) real numbers.

Do NOT leave trig functions in your answers.

(a) \( (2 + 4i)/(1 - 7i) \)

(b) \( (2\sqrt{3} + 2i)^{20} \)

(c) \( e^{3 + i\pi/2} \)
4. (20 pts.) (a) Determine whether \( \int_0^\infty e^{-x} \, dx \) is convergent or divergent.

(b) Use part (a) and the comparison theorem to determine whether \( \int_0^\infty \frac{e^{-x}}{2 + \sin x} \, dx \) is convergent or divergent.

5. (30 pts.) Evaluate the following integrals.

(a) \( \int \frac{\ln x}{x^2} \, dx \)

(b) \( \int \cos x \cos(3x) \, dx \)

(c) \( \int \frac{dx}{x^2 \sqrt{1 - x^2}} \)
6. (10 pts.) Write out the partial fraction decomposition of the function \( \frac{x}{x^2 - 1} \).

7. (15 pts.) Solve the differential equation \( \frac{dy}{dx} = \frac{x^2 + 1}{x^2 y} \) with the initial condition \( y(1) = -2 \) for \( y \) as a function of \( x \); that is, find \( y(x) \).

8. (15 pts.) Use Euler’s method with step size 1.0 to estimate \( x(3.0) \), where \( x(t) \) is the solution of the initial value problem

\[
\frac{dx}{dt} = x + t, \quad x(0) = 0.
\]
9. (20 pts.) Consider the integral \( I = \int_0^2 \frac{2}{4x + 1} \, dx \).

(a) Use the Midpoint rule with \( n = 2 \) subintervals to approximate \( I \).

(b) How large should \( n \) be so that the midpoint approximation of \( I \) is accurate to within \( 6 \times 10^{-4} \)?