Problem set 2

Do for Monday, January 22

Rudin, p. 21: #4, #5
Rudin, p. 43: #2, #3, #4, #7, #9 (a,b,c)

Also:

1. 
   (a) Show that if \( \{p_n\} \) is a Cauchy sequence of rational numbers, then the set \( \{p_1, p_2, \ldots\} \) is bounded from above and below.
   
   (b) Show that if \( \{p_n\} \) and \( \{q_n\} \) are Cauchy sequences of rational numbers, then \( \{p_n q_n\} \) is again a Cauchy sequence.
   
   (c) Let \( \{p_n\}, \{p'_n\}, \{q_n\}, \{q'_n\} \) be Cauchy sequences of rational numbers such that \( \{p_n\} \sim \{p'_n\} \) and \( \{q_n\} \sim \{q'_n\} \). Show that \( \{p_n q_n\} \sim \{p'_n q'_n\} \). (Recall that if \( \{q_n\} \) and \( \{q'_n\} \) are Cauchy sequences of rational numbers, then \( \{q_n\} \sim \{q'_n\} \) means \( \lim_{n \to \infty} (q_n - q'_n) = 0 \))

This shows that multiplication of real numbers (as identified with equivalence classes of Cauchy sequences of rational numbers) is well defined.