Problem set 3

Do for Friday, April 20

Rudin, p. 114 # 11, # 12 (Straightforward)
Rudin, p. 138: #2, #5, #8

Also:

1. For any \( n \), find the Taylor polynomial \( P(t) \) at the point \( \alpha = 0 \), for the function \( f(x) = \frac{1}{1 + x} \). (See Theorem 5.15 for notation.)

2. Prove the following inverse function theorem:
   Suppose that \( f \) is a real valued differentiable function on \((0, 1)\) with \( f' \) continuous and \( f'(x_0) > 0 \) for some \( x_0 \in (0, 1) \). Prove that there is an interval \((a, b)\) containing \( x_0 \) so that \( f : (a, b) \to (c, d) \) is one to one and onto another interval \((c, d)\). Show also that if \( g : (c, d) \to (a, b) \) is defined by \( g(f(x)) = x \), then \( g'(f(x)) = 1/f'(x) \) for all \( x \in (a, b) \).