1. Let $A : \mathbb{R}^3 \to \mathbb{R}^3$ be given by the matrix \[
\begin{pmatrix}
4 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{pmatrix}
\] and suppose $\|B - A\| < 1$. Show that $B$ is invertible.

2. If $A : \mathbb{R}^n \to \mathbb{R}^n$ satisfies $\|I - A\| < 1/3$, what can you say about $\|A^{-1}\|$?

   Hint: Sum the series.

3. Suppose that $[a_{ij}]$ is an $n \times n$ matrix and $A : \mathbb{R}^n \to \mathbb{R}^n$ is given by $Ax = [a_{ij}]x$ for $x \in \mathbb{R}^n$. Let $|A| = (\sum_{ij} a_{ij}^2)^{1/2}$. Show that

   (i) $\|A\| \leq |A|$, and

   (ii) $|A| \leq n\|A\|

   Comment: The above inequalities show that in a sense the norm $\|A\|$ is equivalent to the Euclidean norm $|A|$.

4. (Optional) Suppose that $S$ is a metric space, $X$ a Banach space, and $p \mapsto A(p)$ a continuous mapping from $S$ into $B(X)$, the linear mappings from $X$ to itself satisfying $\|B\| < \infty$. Suppose also that $\|A(p) - I\| = \lambda < 1$ for all $p \in S$. Show that $p \mapsto A(p)^{-1}$ is also continuous.