Problem set 7

Do for Tuesday, May 23

p. 332 #1, # 2, , #8
Also:

1. Use the construction in Theorem 11.20 to find a sequence of simple functions $s : \mathbb{R} \to \mathbb{R}$ that converge to the function $f(x) = x^2$ on $\mathbb{R}$. Calculate $\int_{-2}^{2} x^2 dx$ directly from this sequence. (If, as is likely, you find an expression that you cannot evaluate, try to interpret it as a Riemann sum and use the formula for the Riemann integral to evaluate it.)

2. Recall that $B$ is the $\sigma$-ring generated by all open subsets of $\mathbb{R}^p$. Let $E_0 \subset \mathbb{R}^p$ be a Lebesgue measurable set (i.e. $E_0 \in \mathcal{M}(m)$) and let $N := \{E_0 \cap E : E \in \mathcal{M}(m) \}$. Prove, carefully, that $N$ is a $\sigma$-ring and that a function $f : E_0 \to \mathbb{R}$ is $N$ measurable (according to Definition 11.13) if and only if $f^{-1}(B) \in N$ for all $B \in B$. 
