Problem set 1
Do for Friday, September 29.
Starred problems are to written up carefully and handed in.
Folland p. 117: #1*, #3* #5, #6 # 7*, #8*, #11
Also:
1. Show, by using facts from section 0.6, that if $X$ is a metric space, then the set of all open subsets of $X$ forms a topology for $X$.

2*. Prove that if $\bigcup I_\alpha$ is a disjoint union of open intervals in $\mathbb{R}$, then for all but countably many $\alpha$, $I_\alpha$ is empty. Use this to show that any open set in $\mathbb{R}$ is a countable union of open intervals (some of which may be empty).

   Hint: In each nonempty $I_\alpha$, you can choose a rational number. Show that there is a 1-1 mapping of the set $\{\alpha : I_\alpha \neq \emptyset\}$ into the rational numbers. (This is not the method I suggested in class, but it is easier.)

3* Give an example of a topology which is NOT first countable, and prove that your example works.