Practice Problems for Exam 2
Math 20D, F07

1. Solve the initial value problem

\[ y'' + 16y = 0, \quad y\left(\frac{\pi}{2}\right) = -10, \quad y'\left(\frac{\pi}{2}\right) = 3. \]

What is the long-term behavior of the solution?

2. (a) Find a fundamental set of solutions of the differential equation

\[ y'' - 6y' + 9y = 0. \]

(b) Find the general solution of the differential equation

\[ y'' - 6y' + 9y = 2e^{3t}. \]

3. Consider the differential equation

\[ (1 - x)y'' + xy' - y = 0, \quad 0 < x < 1. \]

(a) Verify that \( y_1 = e^x \) is a solution.

(b) Find a second solution \( y_2 \) which is linearly independent from \( y_1 \). Verify that the two solutions are linearly independent.

(c) Find the general solution of the differential equation

\[ (1 - x)y'' + xy' - y = g(x), \quad 0 < x < 1, \]

where \( g \) is an arbitrary continuous function. Your answer will contain integrals.

4. Consider the system

\[ \vec{x}' = \begin{pmatrix} \alpha & 2 \\ -2 & 0 \end{pmatrix} \vec{x}. \]
(a) Determine the eigenvalues in terms of $\alpha$.

(b) Find the critical value or values of $\alpha$ where the qualitative nature of the phase portrait of the system changes.

(c) Find the general solution of the system and sketch the phase portrait in each of the following cases: $\alpha = 4, \alpha = 0, \alpha = 5$.

(d) Solve the initial value problem

$$\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

in the case $\alpha = 5$. How does the solution behaves as $t \to \infty$?