1. (i) State the order of the given differential equation. (ii) Classify the differential equation as ordinary or partial and (iii) linear or nonlinear.

   (a) \( \frac{d^2 y}{dt^2} + \sin(t + y) = \cos t \)

   (b) \( \frac{\partial^3 y}{\partial t^3} + \frac{\partial y}{\partial t} = 2e^t y + t \)

2. Determine whether each of the given functions is a solution of the differential equation:

   \( t^2 y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2} \ln(t), \quad y_2(t) = 2t^{-4} \)

3. Find the solution of the given initial value problem in explicit form:

   \( y' - y = 2te^{2t}, \quad y(0) = 1 \)

4. Find the solution of the given initial value problem in explicit form. Also determine the interval on which the solution is valid:

   \( y' = (1 - 2x)y^2, \quad y(0) = -\frac{1}{6} \)

5. Sketch several solutions to the given autonomous differential equation in the \( t-y \) plane. Determine the equilibrium solutions and classify each as asymptotically stable, unstable, or semistable.

   \( \frac{dy}{dt} = (y^2 - 16)(y^2 - 4y + 4) \)

   Are there any initial conditions which guarantee that a solution will remain finite in the long-term?

6. Solve the given differential equation:

   \( y' = -\frac{2x + 3y}{3x + 4y} \)
7. Find the general solution of the given differential equation:
   (a) \(9y'' - 12y' + 4y = 0\)
   (b) \(9y'' + 6y' + 82y = 0\)
   (c) \(y'' - 3y = 0\)

8. Determine a suitable form for the particular solution \(Y_p\) of the following given differential equation or system if you were to solve it using the method of undetermined coefficients. (Don’t solve though.)
   (a) \(y'' - 3y' - 4y = 2e^{-t} + t \cos(2t)\)
   (b) \(\vec{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}\)

9. Consider the linear system

\[
\begin{align*}
\frac{dx}{dt} &= x + 2y \\
\frac{dy}{dt} &= -5x - y
\end{align*}
\]

Write the system in matrix form, solve the system, and sketch the phase portrait.

10. Recall that Newton’s Law of Cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the ambient temperature.

   (a) Suppose that the ambient temperature is 15°C and the rate constant is 0.1 (hr)\(^{-1}\). Write the differential equation implied by Newton’s Law of Cooling.

   (b) Solve the differential equation given that the initial temperature of the object is 21°C.
11. Find the recurrence relation satisfied by the coefficients $a_n$ of the power series solution of the following differential equation about the ordinary point $x_0 = 0$. Also find the first four terms in each of two linearly independent solutions.

$$y'' - xy' - y = 0.$$ 

12. What can you say about the radius of convergence of series solutions about each given point $x_0$ for the given differential equation?

$$(x^2 - 2x + 5)y'' + xy' + 5y = 0, \quad x_0 = 1, \quad x_0 = 0$$

13. Use the Laplace transform to solve the given IVP.

(a) 

$$y'' - y' = \cos(2t) + u_6(t) \cos(2t - 12), \quad y(0) = -4, \quad y'(0) = 0$$

(b) 

$$y'' + 3y' + 2y = \delta(t - 5) + u_{10}(t), \quad y(0) = 0, \quad y'(0) = 1/2$$

14. If the Wronskian of $f$ and $g$ is $3e^{4t}$ and $f(t) = e^{2t}$, find $g(t)$. 