Let $n \in \mathbb{N}^\neq 0$ and $a, b, c \in \mathbb{Z}$.

(1) Prove that $a \equiv a \mod n$.

(2) Prove that if $a \equiv b \mod n$ then $b \equiv a \mod n$.

(3) Prove that if $a \equiv b \mod n$ and $b \equiv c \mod n$ then $a \equiv c \mod n$. 
For all three parts, recall that the definition of $a \equiv b \mod n$ is that $a$ and $b$ have the same remainders when divided by $n$.

Alternatively, we could use the equivalent characterization that $a \equiv b \mod n$ if and only if $n|(a-b)$.

1. Prove that $a \equiv a \mod n$.
   
   * $a$ has the same remainder as itself when divided by $n$. Alternatively, $n|0$ because all integers divide 0.

2. Prove that if $a \equiv b \mod n$ then $b \equiv a \mod n$.
   
   The assumption says that $a$ and $b$ have the same remainders when divided by $n$. But, that means that $b$ and $a$ have the same remainders when divided by $n$. So $b \equiv a \mod n$.
   
   Alternatively, the assumption is equivalent to $n|(a-b)$. So, there is $C \in \mathbb{Z}$ such that $Cn = a - b$. Multiplying by $-1$ gives $-Cn = b - a$ so $n|(b-1)$.

3. Prove that if $a \equiv b \mod n$ and $b \equiv c \mod n$ then $a \equiv c \mod n$.
   
   The assumption says that $a$ and $b$ have the same remainders when divided by $n$, say $r$. Also that $b$ and $c$ have the same remainders when divided by $n$, say $s$. Then $r$ and $s$ are both the remainder obtained when we divide $b$ by $n$. But, this remainder is unique so $r = s$. Thus, $a$ and $c$ have the same remainder when divided by $n$.
   
   Alternatively, the assumption gives that $n|(a - b)$ and $n|(b - c)$. So, there are $\alpha, \beta \in \mathbb{Z}$ such that $\alpha n = a - b$ and $\beta n = b - c$. We can rewrite the second equation as $b = \beta n + c$ and substitution in the first equation gives $\alpha n = a - (\beta n + c)$. Grouping terms, we get $(\alpha + \beta)n = a - c$. Thus $n|(a - c)$. 