(c) Compute $C(a)$ if $G = S_3$ and $a = (1, 2, 3)$.

Since $(1, 2)(1, 2, 3) = (2, 3) \neq (1, 3) = (1, 2, 3)(1, 2)$, $(1, 3)(1, 2, 3) \neq (1, 2, 3)(1, 3)$, and $(2, 3)(1, 2, 3) \neq (1, 2, 3)(2, 3)$, we see that $(1, 2)$, $(1, 3)$, and $(2, 3)$ do not belong to $C((1, 2, 3))$. Since $\langle a \rangle \subseteq C(a)$ we then have $C((1, 2, 3)) = \{(1), (1, 2, 3), (1, 3, 2)\}$.

(d) Compute $C(a)$ if $G = S_3$ and $a = (1, 2)$.

Since $(1, 2)(1, 2, 3) \neq (1, 2, 3)(1, 2)$, $(1, 2)(1, 3, 2) \neq (1, 3, 2)(1, 2)$, $(1, 2)(1, 3) \neq (1, 3)(1, 2)$, and $(1, 2)(2, 3) \neq (2, 3)(1, 2)$, we have $C((1, 2)) = \{(1), (1, 2)\}$.

(c) Compute the center of $S_3$.

Since $C((1, 2, 3)) = \{(1), (1, 2, 3), (1, 3, 2)\}$ and $C((1, 2)) = \{(1), (1, 2)\}$, by Exercise 19 we have $Z(S_3) = \{(1)\}$.

4. Give the subgroup diagram of $\mathbb{Z}_{60}$.

\[
\begin{array}{cccc}
& & & \\
& \mathbb{Z}_{60} & & \\
3\mathbb{Z}_{60} & 5\mathbb{Z}_{60} & 2\mathbb{Z}_{60} & \\
& & \times & & \\
15\mathbb{Z}_{60} & 6\mathbb{Z}_{60} & 10\mathbb{Z}_{60} & 4\mathbb{Z}_{60} & \\
& \times & & \times & & \\
30\mathbb{Z}_{60} & 12\mathbb{Z}_{60} & 20\mathbb{Z}_{60} & \\
& \times & & \times & & \\
& & & (0)
\end{array}
\]

14. Prove that any cyclic group with more than two elements has at least two different generators.

Let $G = \langle a \rangle$ be such that $|G| > 2$. Hence $o(a) \neq 1$, and so $a \neq a^{-1}$. Clearly $G = \langle a \rangle = \langle a^{-1} \rangle$, and so $G$ has at least two different generators.

20. Let $G$ be a group with $p^k$ elements, where $p$ is a prime number and $k \geq 1$. Prove that $G$ has a subgroup of order $p$.

Let $a \in G$, with $a \neq e$. Thus $o(a) = p^t$, where $t \geq 1$, since $1 \neq o(a) \mid |G|$. Let $b = a^{p^{-1}}$. Since $b \neq e$, but $b^p = (a^{p^{-1}})^p = a^p = e$ we have $o(b) = p$. Hence $\langle b \rangle$ is a subgroup of $G$ of order $p$. 
