**Exercise (II.12).** Suppose that $A \subseteq \mathbb{Z}$. Write the following statement entirely in symbols using the quantifiers $\forall$ and $\exists$. Write out the negative of this statement in symbols.

There is a greatest number in the set $A$.

Give an example of a set $A$ for which this statement is true. Give an example of a set $A$ for which it is false.

---

**Exercise (II.17).** Functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined as follows.

$$f(x) = \begin{cases} 
  x + 2 & \text{if } x < -1, \\
  -x & \text{if } -1 \leq x \leq 1 \\
  x - 2 & \text{if } x > 1 
\end{cases} \quad g(x) = \begin{cases} 
  x - 2 & \text{if } x < -1, \\
  -x & \text{if } -1 \leq x \leq 1 \\
  x + 2 & \text{if } x > 1 
\end{cases}$$

Find the functions $f \circ g$, $g \circ f$. Is $g$ the inverse of $f$? Is $f$ injective or surjective? Is $g$?
Exercise (II.18).

(a) Suppose that $f : X \to Y$ and $g : Y \to Z$ are surjections. Prove that the composite $g \circ f : X \to Z$ is a surjection.

(b) Suppose that $f : X \to Y$ and $g : Y \to Z$ are injections. Prove that the composite $g \circ f : X \to Z$ is an injection.