True or False?

(a) There is a set of sentences $A$ such that $Cn A = \text{Th } \mathcal{N}$.
(b) There is a representable set of sentences $A$ such that $Cn A = \text{Th } \mathcal{N}$.
(c) There is a recursive set of sentences $A$ such that $Cn A = \text{Th } \mathcal{N}$.
(d) There is an arithmetical set of sentences $A$ such that $Cn A = \text{Th } \mathcal{N}$.
(e) There is a $\Sigma_1$ set of sentences $A$ such that $Cn A = \text{Th } \mathcal{N}$.
Recall that a set of sentences is said to be representable (resp., recursive, arithmetical, or $\Sigma_1$) if the set of Gödel numbers of sentences in the set is representable (resp., recursive, arithmetical, or $\Sigma_1$). Note that we have the following inclusions and equalities:

\[
\text{representable sets} \subseteq \text{recursive sets} \subsetneq \text{arithmetical sets} = \text{definable sets}.
\]

Tarski’s undefinability theorem states that $\text{Th } \mathcal{N}$ is not definable in $\mathcal{N}$. Therefore, (by the set properties above) items (b), (c), (d), (e) are all false. Item (a), however, is true because we can take the set of sentences $A$ to be $\text{Th } \mathcal{N}$. Then $Cn A = A$ because a theory is closed under logical implication.