 Assigned reading: Chapters 12-13 of Gallian.

Recommended practice questions: Chapter 12 of Gallian, exercises
30, 31, 32, 33
Chapter 13 of Gallian, exercises
1, 2, 5, 6, 9, 15, 18

Assigned questions to hand in:

(1) (Gallian Chapter 12 # 43) Let \( R = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \) and \( S = \{(a, b, c) \in R : a + b = c\} \). Prove or disprove that \( S \) is a subring of \( R \).

(2) (Gallian Chapter 12 #44, special case) Suppose that \( a^2 = a \) for all elements \( a \) of some ring. Show that \( -a = a \) for all \( a \) in the ring. Bonus: can you generalize this from \( n = 2 \) to any positive even integer?

(3) (Gallian Chapter 13 #10) Describe all zero-divisors and units of \( \mathbb{Z} \oplus \mathbb{Q} \oplus \mathbb{Z} \).

(4) (Gallian Chapter 13 #28) Let \( R \) be the set of all real-valued functions defined for all real numbers under function addition and multiplication.
\( \text{(a) } \) Determine all zero-divisors of \( R \).
\( \text{(c) } \) Show that every nonzero elements is a zero-divisor or a unit.

(5) (Gallian Chapter 13 #54) Let \( R \) be a ring with \( m \) elements. Show that the characteristic of \( R \) divides \( m \).

(6) (Gallian Chapter 13 #32) Let \( R = \{0, 2, 4, 6, 8\} \) under addition and multiplication modulo 10. Prove that \( R \) is a field.

(7) (Gallian Chapter 13 #60) In a commutative ring of characteristic 2, prove that the idempotents form a subring. Recall (from question 18) that \( a \) is idempotent if \( a^2 = a \).