MATH 103B Homework 3
DUE April 19, 2013 VERSION April 12, 2013

Assigned reading: Chapters 13-14 of Gallian.

Recommended practice questions: Chapter 13 of Gallian, exercises
35, 45, 47, 49, 51, 62, 63
Chapter 14 of Gallian, exercises
8, 9, 12, 13, 17, 24, 28, 29

Assigned questions to hand in:

1. (Gallian Chapter 13 # 46) Suppose that $a$ and $b$ belong to a commutative ring and $ab$ is a zero-divisor. Show that either $a$ or $b$ is a zero-divisor.

2. (Gallian Chapter 13 # 48) Suppose that $R$ is a commutative ring without zero-divisors. Show that the characteristic of $R$ is zero or prime.

3. (Gallian Chapter 14 # 4) Find a subring of $\mathbb{Z} \oplus \mathbb{Z}$ that is not an ideal of $\mathbb{Z} \oplus \mathbb{Z}$. Justify your answer.

4. (Gallian Chapter 14 # 10) If $A$ and $B$ are ideals of a ring, show that the sum of $A$ and $B$, $A + B = \{a + b : a \in A, b \in B\}$, is an ideal.

5. (Gallian Chapter 14 # 15) If $A$ is an ideal of a ring $R$ and 1 belongs to $A$, prove that $A = R$.

6. (Gallian Chapter 14 # 22) Let $I = \langle 2 \rangle$. Prove that $I[x]$ is not a maximal ideal of $\mathbb{Z}[x]$, even though $I$ is a maximal ideal of $\mathbb{Z}$.

7. (Gallian Chapter 14 # 26) If $R$ is a commutative ring with unity and $A$ is a proper ideal of $R$, show that $R/A$ is a commutative ring with unity.

8. (Gallian Chapter 14 # 35) In $\mathbb{Z} \oplus \mathbb{Z}$, let $I = \{(a, 0) : a \in \mathbb{Z}\}$. Show that $I$ is a prime ideal but not a maximal ideal.