1.3 #1 for 3 Rewrite each of these tautologies using the conventions of the present section to minimize the number of parentheses.

\[ ((\neg(\neg A)) \leftrightarrow A) : \quad \neg\neg A \leftrightarrow A \]
\[ ((\neg(A \rightarrow B)) \leftrightarrow (A \land (\neg B))) : \quad \neg(A \rightarrow B) \leftrightarrow A \land \neg B \]
\[ ((\neg(A \leftarrow B)) \leftrightarrow ((A \land (\neg B)) \lor ((\neg A) \land B))) : \quad \neg(A \leftarrow B) \leftrightarrow (A \land \neg B) \lor (\neg A \land B) \]
\[ ((\neg(A \lor B)) \leftrightarrow ((\neg A) \lor (\neg B))) : \quad \neg(A \lor B) \leftrightarrow \neg A \lor \neg B \]
\[ ((\neg(A \lor B)) \leftrightarrow ((\neg A) \land (\neg B))) : \quad \neg(A \lor B) \leftrightarrow \neg A \land \neg B \]

1.3 #2 Give an example of wffs \( \alpha \) and \( \beta \) and expressions \( \gamma \) and \( \delta \) such that

\( (\alpha \land \beta) = (\gamma \land \delta) \quad \text{but} \quad \alpha \neq \gamma. \)

**Solution:** Let
\[ \alpha = (A \land B) \quad \beta = C \quad \gamma = (A \lor \delta = B) \land C. \]

Then \( \alpha, \beta \) are wffs and
\[ (\alpha \land \beta) = ((A \land B) \land C) = (\gamma \land \delta) \]

but \( \alpha \neq \gamma. \)

1.5 #1a Let \( G \) be the following three-place Boolean function

\[ G(F, F, F) = T \quad G(T, F, F) = T \]
\[ G(F, F, T) = T \quad G(T, F, T) = F \]
\[ G(F, T, F) = T \quad G(T, T, F) = F \]
\[ G(F, T, T) = F \quad G(T, T, T) = F. \]

Find a wff, using at most the connectives \( \land, \lor, \) and \( \neg, \) that realizes \( G. \)

**Solution:**
\[ (\neg A \land \neg B \land \neg C) \lor (A \land \neg B \land \neg C) \lor (\neg A \land \neg B \land C) \lor (\neg A \land B \land \neg C) \]

1.5 #3 Show that \{\neg, \#\} is not complete.

**Solution:** The constant unary connectives are not representable using this set of connectives. Formally, we prove by induction on the structure of formulas that any unary wff, \( \alpha, \) using only these connectives \( B_\alpha(X) \) is not constant.

- (Base case) If \( \alpha = A \) then \( B_\alpha(X) = X, \) the identity function.
- (Inductive step) Suppose true for \( \alpha_1, \alpha_2, \alpha_3 \) (so \( \alpha_i \) has \( A \) as its only sentence symbol and \( B_{\alpha_i}(X) \) is not a constant function).
  - If \( \alpha = \neg \alpha_1 \) then \( B_\alpha = \text{the opposite of } B_{\alpha_1}. \) Since \( B_{\alpha_1} \) is not constant, neither is \( B_\alpha. \)
  - Suppose \( \alpha = (\# \alpha_1 \alpha_2 \alpha_3). \) By the hypothesis on the \( \alpha_i, \) for each \( i, \) either \( \bar{v}(\alpha_i) = v(A) \) or \( \bar{v}(\alpha_i) = \bar{v}(\neg A). \) Since the truth value of \( (\# \alpha_1 \alpha_2 \alpha_3) \) only depends on the number of \( \alpha_i \) assigned \( T/F \) (rather than which is which), we can split into two cases:
    - At least two of the \( \alpha_i \) have \( \bar{v}(\alpha_i) = v(A) \). In this case, \( \bar{v}(\alpha) = v(A), \)
      and so \( B_\alpha \) is not constant.
    - At least two of the \( \alpha_i \) have \( \bar{v}(\alpha_i) = \bar{v}(\neg A). \) In this case, \( \bar{v}(\alpha) = \bar{v}(\neg A), \)
      so \( B_\alpha \) is also not a constant function.
1.5 #9a Find a formula in CNF that is tautologically equivalent to $A \leftrightarrow B \leftrightarrow C$.

$$(A \lor \neg B) \land (A \lor \neg C) \land (B \lor \neg A) \land (B \lor \neg C) \land (C \lor \neg A) \land (C \lor \neg B)$$