Math 171B Homework Assignment #4

Due Date: May 6, 2016

1. (5 points) Sketch the contours of the function
\[ x_1^2 + 4x_2^2 - 4x_1 - 8x_2. \]
Determine the point \( x^* \) which minimizes \( f(x) \).

2. (5 points) Find all the local minimizers of the function
\[ 1 + x + x^2 + x^3 + x^4. \]

3. (10 points) Show that the function
\[ (x_2 - x_1^2)^2 + x_1^5 \]
has only one stationary point, which is also a saddle point.

4. (10 points) Find the global minimizer of the function
\[ 2x_1^2 + x_2^2 - 2x_1x_2 + 2x_1^3 + x_1^4. \]

5. (10 points) Let \( H \in \mathbb{R}^{n \times n} \) be a symmetric positive definite matrix and \( g \in \mathbb{R}^n \) be vector. Express the global minimum value of \( g^T x + \frac{1}{2} x^T H x \) in terms of \( g, H \).

6. (10 points) Let \( f(x) = (x_1 - x_2^2)^2 - x_1^2 x_2 \). Show that for every \( 0 \neq d \in \mathbb{R}^2 \), the zero \( 0 \) is a local minimizer of the univariate function \( g(t) := f(td) \) in \( t \in \mathbb{R} \). Decide whether or not the origin \( (0,0) \) is a local minimizer of \( f \).