1. We look for \( \frac{1}{n} < \frac{5}{12} < \frac{1}{(n-1)} \). So \( n = 3 \). Now \( \frac{5}{12} - \frac{1}{3} = \frac{1}{12} \). So the answer is \( \frac{5}{12} = \frac{1}{3} + \frac{1}{12} \). For \( \frac{4}{11} \), \( n = 3 \) again. \( \frac{4}{11} - \frac{1}{3} = \frac{1}{33} \) so \( 4/11 = 1/3 + 1/33 \). As for the Babylonian sexagesimal multiply 5 by 60 and divide by 12 and get 25 so \( \frac{5}{12} = 25/60 = 25 \). For \( \frac{4}{11} \) multiply 4 by 60 and get 240. Divide by 11 and get 21 with remainder 9. Multiply 9 by 60 and get 540 divide by 11 and get 49 with remainder 1. Multiply by 60 and divide by 11 and get 5 with remainder 5. Multiply 5 by 60 and divide by 11 and get 27 with remainder 3. Multiply 3 by 60 and divide by 11 and get 16 with remainder 4. Thus the expansion is \( ; 21, 49, 5, 27, 16, 21, 49, 5, 27, 16, \ldots \)

2. \( \frac{31}{3} \) is \( 1 + 2 + 4 + 8 + 16 \). So we have

\[
\begin{align*}
37 & \quad 1 \\
74 & \quad 2 \\
148 & \quad 4 \\
296 & \quad 8 \\
592 & \quad 16
\end{align*}
\]

so \( 31 \) times 37 is \( 37 + 74 + 148 + 296 + 592 = 1147 \). \( 12 = 4 + 8 \) so 37 times 12 is \( 148 + 296 = 444 \).

3. Since we are asking for \( 2/3 \) of the original quantity at first let’s try following the recipe “plugging in” \( 3 \). \( 2/3 \) of 3 is 2 and the new quantity is 5. Subtracting \( \frac{1}{3} \) of this is subtracting \( 1 + 2/3 \) (a legal Egyptian fraction). This subtracted from 5 is \( 3 + 1/3 \). If we multiply this by 3 we get \( 9 + 1 = 10 \). Thus the solution is 9.

4. Recall that \( (1 + x + x^2 + \ldots + x^n)(x - 1) = x^{n+1} - 1 \). Thus if \( x = 3 \) and \( n = 15 \). We are looking at

\[
\frac{40346721 - 1}{3 - 1} = \frac{40346720}{2} = 20173360. \]

5. \( P(n) \) is the statement that there is no \( m \) so that \( n^2 = 3m^3 \). Suppose \( P(n) \) is false. We must show that this would imply that there is an \( m \) with \( 1 < m < n \) so that \( P(m) \) is false. Then \( n > 1 \) since 1 is not divisible by 3. Now since \( P(n) \) is false there is \( m \) such that \( n^2 = 3m^2 \). We note that \( m > 1 \) (otherwise \( n^2 = 3 \)) and \( m < n \) since if \( m \geq n \) then \( 3m > m \geq n \) so \( 3m^2 > mn \geq n^2 \). Now 3 divides \( n^2 \) so according to Euclid since 3 is prime 3 divides \( n \). Thus \( n = 3k \). So we have an identity

\[
9k^2 = 3m^2.
\]

This implies that \( m^2 = 3k^2 \) this implies that \( P(m) \) is false.