1. A field $F$ is said to be \textit{totally real} if the only pair $(a, b)$ with $a, b \in F$ satisfying $a^2 + b^2 = 0$ is $(0, 0)$. Show that if $F$ is a field then the set of matrices in $M_2(F)$ (the two by two matrices over $F$)

$$R = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a, b \in F \right\}$$

is a subring with identity in $M_2(F)$ and it is a field if and only if $F$ is totally real.

2. In (a),(b),(c) below $R$ and $S$ are rings and $\phi : R \rightarrow S$ is a map. Determine which $\phi$ are ring homomorphisms.

(a) $R = S = \mathbb{Z} (= J)$ and $\phi(n) = n^2$.

(b) $L, M$ rings, $f : L \rightarrow M$ a ring homomorphism, $x$ an indeterminate and $R = L[x], S = M[x]$ and $\phi(a_0 + a_1x + \ldots + a_nx^n) = f(a_0) + f(a_1)x + \ldots + f(a_n)x^n$ for $a_0, \ldots, a_n \in L$.

(c) $R = \mathbb{Z}_3(= J_3)$ and $\phi([m]) = [m^3]$.

3. Let $R = \{2n \mid n \in \mathbb{Z}\}$ show that $R$ is not a principal ideal domain.

4. Let $F$ denote the field of rational numbers. Calculate a greatest common divisor of the polynomials $f(x) = 1 + 4x + 4x^2 + 3x^3$ and $g(x) = 1 + x^2 + x^4$. 