1. Draw a picture of the parallelogram in the $xy$ plane with vertices 
   
   $(2, 4), (4, 6), (4, 5), (6, 7)$.

   Calculate its area.

   Solution:

   We note that the adjacent sides at the vertex $(2, 4)$ have displacement vectors 
   $2\mathbf{i} + 2\mathbf{j}$ ($(2, 4)$ to $(4, 6)$ and $2\mathbf{i} + 3\mathbf{j}$ ($(2, 4)$ to $(4, 5)$). Thus the area is the norm of

   $\begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 2 & 3 & 0 \end{vmatrix} = 2\mathbf{k}$.

   So the answer is 2.

2. Which of the following two limits exist? If the limit exists calculate it. In either case you must justify your answer.

   a) $\lim_{(x,y,z) \to (0,0,0)} \frac{xy+z^3}{1+x^2+y^2+z^2}$.

   b) $\lim_{(x,y,z) \to (0,0,0)} \frac{xy+z^3}{x^2+y^2+z^2}$.

   Solution:

   a) The limit exists since the function is a ratio of two polynomials and the denominator is not 0 at $(0, 0, 0)$ this implies the limit is the value at $(0, 0, 0)$ which is 0.

   b) If we look at the line $x = y$ and $z = 0$ then the value of the function is $\frac{x^2}{2x^2} = \frac{1}{2}$. If we look at the line $y = z = 0$ then the value is $0 \frac{z^2}{x^2} = 0$. Thus the limit does not exist.

3) Assume that $f$ is a function on $\mathbb{R}^3$ that is differentiable at $(1,1,-1)$ and
\[
\frac{\partial f}{\partial x}(1,1,-1) = -1, \quad \frac{\partial f}{\partial y}(1,1,-1) = 2, \quad \frac{\partial f}{\partial z}(1,1,-1) = -1.
\]

Suppose that \(u(t) = \cos t, v(t) = 1 + t, w(t) = -e^t\). Calculate the derivative of \(f(u(t), v(t), w(t))\) in \(t\) at \(t = 0\).

Solution:

\(u(0) = 1, v(0) = 2\) and \(w(0) = -1\). \(u'(0) = \sin(0) = 0, v'(0) = -1, w'(0) = -1\) thus if \(g(t) = f(u(t), v(t), w(t))\) then

\[
g'(0) = \frac{\partial f}{\partial x}(u(0), v(0), w(0))u'(0) + \frac{\partial f}{\partial y}(u(0), v(0), w(0))v'(0) + \frac{\partial f}{\partial z}(u(0), v(0), w(0))w'(0)
\]

\[
= (-1)0 + 2(-1) + (-1)(-1) = -1.
\]

4. Calculate the arc length of the following two paths curves.

a) \((t, t^2, \frac{2t}{3}), \ 0 \leq t \leq 1\).

b) \((\cos|t|, \sin|t|, t), \ -\pi \leq t \leq \pi\). (Hint: You will have to divide the path into two pieces.)

Solutions:

a) The velocity vector is \((1, 2t, t^2)\) thus the arclength is

\[
\int_0^1 \|(1, 2t, t^2)\| \, dt = \int_0^1 \sqrt{1 + 4t^2 + 4t^4} \, dt
\]

\[
= \int_0^1 (1 + 2t^2) \, dt = 1 + \frac{2}{3} = \frac{5}{3}.
\]

b) You must calculate the length of the path \((\cos(-t), \sin(-t), t)\) on the interval \(-\pi \leq t \leq 0\) and add it to the length of the path \((\cos t, \sin t, t)\) on the interval \(0 \leq t \leq \pi\). The first is

\[
\int_{-\pi}^0 \sqrt{\sin^2(-t) + \cos^2(-t) + 1} \, dt = \sqrt{2} \int_{-\pi}^0 dt = \sqrt{2} \pi.
\]

The second is

\[
\int_0^\pi \sqrt{\sin^2 t + \cos^2 t + 1} \, dt = \sqrt{2} \int_0^\pi dt = \sqrt{2} \pi.
\]

So the answer is \(\sqrt{2} \pi + \sqrt{2} \pi = 2\sqrt{2} \pi\).