Practice Midterm Math 20e

The purpose of this practice test is to let you know the level of difficulty of the questions. There could be questions involving material that had not been covered as of 10/21.

1. a) Suppose that \( \mathbf{a} \) and \( \mathbf{b} \) are both non-zero vectors and \( \mathbf{a} \cdot \mathbf{b} = 0 \) and suppose that \( c \) is a scalar. Consider the following equations with \( \mathbf{x} \) an unknown vector:

\[
\mathbf{a} \cdot \mathbf{x} = c \\
\mathbf{a} \times \mathbf{x} = \mathbf{b}.
\]

Let \( \theta \) be the angle between \( \mathbf{a} \) and a solution \( \mathbf{x} \). Calculate \( \cos \theta \) and \( \sin \theta \).

b) If in part a) \( \mathbf{a} = (\sqrt{2}, 1, 1), \mathbf{b} = \sqrt{2}(\mathbf{j} - \mathbf{k}), c = 2 \) what is the angle between a solution \( \mathbf{x} \) and \( \mathbf{a} \)? Find all solutions \( \mathbf{x} \).

2. Consider the cone \( z = \sqrt{x^2 + y^2} \). Calculate the tangent plane at every point on the surface except \((0, 0, 0)\). Show that the line through the origin in the direction of the position vector \((x, y, z)\) to a point on the surface is completely contained in the tangent plane at the point.

3. A function \( f(x, y, z) \) is differentiable at \((1, 1, -1)\) with gradient \((2, 1, 1)\) at \((1, 1, -1)\). Suppose that \( u(t), v(t), w(t) \) are functions of \( t \) that are differentiable at \( t = 1 \), \( u(1) = v(1) = 1 \) and \( w(1) = -1 \) and \( u'(1) = -1, v'(1) = 2, w'(1) = 3 \). Calculate the derivative of \( f(u(t), v(t), w(t)) \) at \( t = 1 \).

4. Calculate the arclength of the path

\[
\mathbf{x}(t) = (\cos t^2, \sin t^2, t^2)
\]

for \( 0 \leq t \leq \pi \).