6.5.4. Suppose $X$ and $Y$ are standard normal variables. Find an expression for $P(X + 2Y \leq 3)$ in terms of the standard normal distribution function $\Phi$,

(a) in case $X$ and $Y$ are independent;
(b) in case $X$ and $Y$ have bivariate normal distribution with correlation 1/2.

6.5.6. Let $X$ and $Y$ be independent standard normal variables.

(a) For a constant $k$, find $P(X > kY)$.
(b) If $U = \sqrt{3}X + Y$, and $V = X - \sqrt{3}Y$, find $P(U > kV)$.
(c) Find $P(U^2 + V^2 < 1)$.
(d) Find the conditional distribution of $X$ given $V = v$.

6.5.8. Let $X_1$ and $X_2$ be two independent standard normal random variables. Define two new random variables as follows: $Y_1 = X_1 + X_2$ and $Y_2 = \alpha X_1 + 2X_2$. You are not given the constant $\alpha$ but it is known that $\text{Cov}(Y_1, Y_2) = 0$. Find

(a) the density of $Y_2$;
(b) $\text{Cov}(X_2, Y_2)$.

6.5.9. Suppose $W$ has the normal $(\mu, \sigma^2)$. Given that $W = w$, suppose $Z$ has normal $(aw + b, \tau^2)$ distribution.

(a) Show the joint distribution of $W$ and $Z$ is bivariate normal, and find its parameters.
(b) What is the distribution of $Z$?
(c) What is the conditional distribution of $W$ given $Z = z$?

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From the Pinsky-Karlin text:

Section 3.1. (pp.81-83): Ex. 1.4, 1.5; Pr. 1.4
Section 3.2. (pp.84-87): Ex. 2.2; Pr. 2.2, 2.5