1. Consider a Markov chain with state space \{0, 1, 2\} and transition matrix

\[
P = \begin{bmatrix}
0 & 1/2 & 1/2 \\
1/3 & 0 & 2/3 \\
1/3 & 1/3 & 1/3 \\
\end{bmatrix}.
\]

Suppose \(P(X_0 = i) = 1/3\) for \(i = 0, 1, 2\).

(a) Compute \(P(X_2 = i|X_0 = 0)\) for \(i = 0, 1, 2\).

(b) Compute \(P(X_0 = 1, X_2 = 2|X_1 = 0)\).

2. A point \((X, Y)\) is picked at random (with the uniform distribution) from the unit square \(\{(x, y): 0 \leq x \leq 1, 0 \leq y \leq 1\}\).

(a) Find \(P(X^2 + Y^2 < 1)\).

(b) Find \(E(X^2 + Y^2)\).

3. The random variable \(X\) has the uniform distribution on \((0, 1)\). Given that \(X = x\), the conditional distribution of the random variable \(Y\) is uniform on \((x, 1)\).

(a) Find \(E(Y|X = x)\) for \(0 < x < 1\).

(b) Use the result of part (a) to compute \(E(Y)\).

(c) Find the marginal density function \(f_Y\).

4. Let \(X\) and \(Y\) be standard normal random variables with \(\rho = \text{Corr}(X, Y) = -.5\).

(a) Find \(\text{Var}(X - Y)\).

(b) Find the conditional expectation \(E(Y|X = 3)\).

5. \(X\) and \(Y\) are random variables with \(\text{Var}(X) = 1, \text{Var}(Y) = 4\). Show that \(1 \leq \text{Var}(X + Y) \leq 9\).