Math 183 - Midterm Review (Highlights)

Instructions:
Be sure to bring your official UCSD student ID to the exams.

1. **Probability of events; Venn diagrams.**
   
   Show that for any two events $A$ and $B$ in a sample space $S$,
   
   $$P(A \cap B) \geq 1 - P(A^c) - P(B^c).$$

2. **Conditional probabilities; Bayes formula.**
   
   During a riot 100 people are arrested on suspicion of looting, and each is given a polygraph test. Supposed that the polygraph is 90% reliable when given to a guilty suspect and 98% reliable when given to someone who is innocent. Suppose that 12 out of the 100 people were involved in the looting, what is the probability that a given suspect is innocent if the polygraph says he’s guilty?

   **Solution.** Let $F = \{\text{polygraph says suspect is guilty}\}$, $E = \{\text{suspect is guilty}\}$ ...

3. **Random variables: discrete, continuous; CDF; density; expectation.**
   
   Rayleigh distribution, named after the physicist Lord Rayleigh while studying waves, has density
   
   $$f(y) = \frac{y}{a^2}e^{-y^2/2a^2}, \quad y > 0$$
   
   where $a > 0$ is a constant. Find $E(Y)$.

4. **Joint distribution; marginal distribution; independence.**
   
   Suppose two components of a machine have independent exponentially distributed lifetimes $T_1$ and $T_2$, with densities
   
   $$f_1(t_1) = \lambda e^{-\lambda t_1}, \quad f_2(t_2) = \mu e^{-\mu t_2}, \quad t_1, t_2 > 0$$
   
   where $\lambda$ and $\mu$ are the parameters.
   
   (a) Write out the joint density of $T_1$ and $T_2$;
   
   (b) Find $P(T_1 > T_2)$. 