Turn in the homework by 5:00pm, in the homework box for your Section in the basement of AP&M. Late homework will not be accepted.

1. Exercise 4 on page 26 in Durrett.
2. Exercise 7 on page 26 in Durrett.
3. Exercise 15 on page 27 in Durrett.
4. Exercise 16 on page 72 in Durrett.
5. Exercise 24 on page 73 in Durrett.
6. Problem 62 on page 77 in Durrett.
7. Problem 81 on page 79 in Durrett.
8. A die is rolled repeatedly until a 6 appears, at which point the experiment stops. Describe a sample space for this experiment. Let $E_n$ denote the event that $n$ rolls are necessary to complete the experiment. What outcomes (elements) in the sample space are contained in $E_n$? What is $(\bigcup_{n=1}^{\infty} E_n)^c$?
9. (a) Let $E, F$ be events, and suppose $P(E) = 0.9$ while $P(F) = 0.8$. Show that $P(E \cap F) \geq 0.7$.
   (b) Prove Bonferroni’s inequality: for any events $E, F$,
   $$P(E \cap F) \geq P(E) + P(F) - 1.$$ 
   (c) Use induction to generalize Bonferroni’s inequality to $n$ events: show that, for all events $E_1, \ldots, E_n$,
   $$P(E_1 \cap E_2 \cap \cdots \cap E_n) \geq P(E_1) + \cdots + P(E_n) - (n - 1).$$ 
   Show also that no number smaller than $n - 1$ can be used in this inequality in general.
10. Let $f_n$ denote the number of ways of tossing a coin $n$ times such that successive heads never appear. Since this condition is automatic when $n < 2$, $f_0 = 1$ and $f_1 = 2$.
    (a) Argue that, for $n \geq 2$,
    $$f_n = f_{n-1} + f_{n-2}.$$ 
    (b) Show, by induction, that $f_n < 2 \cdot \left(\frac{7}{4}\right)^n$ for all $n \geq 0$.
    (c) Conclude that, when tossing a fair coin $n$ times, the probability of never seeing two successive heads tends to 0 exponentially fast as $n \to \infty$. 