Homework 7

Available | Friday, February 19 | Due | Friday, February 26

Turn in the homework by 5:00pm, in the homework box for your Section in the basement of AP&M. Late homework will not be accepted.

1. Exercise 60 on page 112 in Durrett.
2. Exercise 61 on page 112 in Durrett.
3. Exercise 66 on page 113 in Durrett.
4. Exercise 30 on page 188 in Durrett.
5. Exercise 31 on page 188 in Durrett.
6. Exercise 36 on page 188 in Durrett.
7. Exercise 37(a) on page 188 in Durrett.

8. (a) Show that a constant random variable \( Y = c \) is independent from any random variable \( X \).
   (b) If \( X \) and \( Y \) are independent and \( c \in \mathbb{R} \), show that \( X \) and \( cY \) are independent.

9. Let \( X,Y \) be uniform random variables with state space \( \{1,2,\ldots,n\} \); i.e. for each \( k \in \{1,\ldots,n\}, \mathbb{P}(X = k) = \mathbb{P}(Y = k) = \frac{1}{n} \). Suppose that \( X \) and \( Y \) are independent. Show that
   \[ \mathbb{E}[|X - Y|] = \frac{n^2 - 1}{3n}. \]
   [Hint: you may use the facts that
   \[ \sum_{k=1}^{n-1} k = \frac{1}{2}n(n-1), \quad \sum_{k=1}^{n-1} k^2 = \frac{1}{6}n(n-1)(2n-1). \]]