1. Let $\Omega = \{\omega_1, \omega_2\}$ with a subjective probability $P : P(\omega_i) > 0$, $i = 1, 2$.

Consider a single period model with one riskless and one risky asset. Suppose the riskless asset $S^0 = \{S^0_0, S^0_1\}$ satisfies $S^0_t = (1 + r)^t$, $t = 0, 1$ for $r = \frac{1}{9}$, and suppose the risky asset $S^1 = \{S^1_0, S^1_1\}$ is such that $S^1_0 = 5$, $S^1_1(\omega_1) = \frac{20}{3}$, $S^1_1(\omega_2) = \frac{40}{9}$. The discounted value of $S^1$ is given by $S^{\ast,1} = S^1_1/S^0_1$ and $\Delta S^{\ast,1} = S^{\ast,1} - S^1_0$.

Consider the optimal portfolio selection problem

$$
(\text{OP}) \begin{cases}
\text{maximize} \{E^p[u(V_t)] : H \in \mathcal{H}\} \\
\text{subject to} \ V_0 = \nu
\end{cases}
$$

where $\mathcal{H}$ is the set of self-financing trading strategies $H = (H^0, H^1)$ and

$$
V_1 = V_1(H) = H^0 S^0_1 + H^1 S^1_1 = (V_0(H) + H^1 \Delta S^{\ast,1}) S^0_1
$$

is the value of $H$ at time one and $V_0 = V_0(H)$ is the initial value of $H$. Here, the initial wealth $\nu$ is assumed to satisfy $\nu \geq 0$.

For each of the following utility functions, use the risk neutral computational approach to find the solution of $(\text{OP})$.

(a) $u(x) = \ln x$

(b) $u(x) = -\exp(-x)$

(c) $u(x) = \gamma^{-1}x^\gamma$ where $-\infty < \gamma < 1$, $\gamma \neq 0$. 