Quiz 2, Math 251  
2.6-3.4

Name(Print): ID #:

Notice: The use of a calculator, cell phone, or any other electronic device is **NOT** permitted during this quiz. Write Down detailed steps. Partial credits will be given for that.

1. Consider the ODE

\[
\left(\frac{\sin y}{y} - 2e^{-x}\sin x\right)dx + \left(\frac{\cos y + 2e^{-x}\cos x}{y}\right)dy = 0.
\]

a) Show that the given ODE is not exact. (2pts)

**solution:** Compare $M_y$ and $N_x$. Notice that

\[
M_y = \frac{\cos y}{y} - \frac{\sin y}{y^2}; \quad N_x = \frac{2}{y}\left(-e^{-x}\cos x - e^{-x}\sin x\right)
\]

hence $M_y \neq N_x$ and the equation is **NOT** exact.

**Comment:** if one knew compare $M_y$ and $N_x$, but did not correctly get the partial derivative, 1 point off.

b) Show that it becomes exact when multiplied by the following integrating factor. (3pts)

\[
\mu(x, y) = ye^x.
\]

**solution:** Multiplying the given $\mu$ on both sides:

\[
(sin ye^y - 2y\sin x)dx + (cos ye^x + 2\cos x)dy = 0.
\]

For this modified equation:

\[
M_y = (\sin ye^y - 2y\sin x)_y = \cos ye^x - 2\sin x;
\]

\[
N_x = (\cos ye^x + 2\cos x)_x = \cos ye^x - 2\sin x.
\]

Hence $M_y = N_x$ and the modified equation is exact.

**Comment:** Obtaining the modified equation, 1 point; Correctly finding the two partial derivatives, 2 points.

c) Solve the equation. (10 pts)

**solution:** Introduce the potential equation $\phi$ which satisfies the following system

\[
\phi_x = \sin ye^y - 2y\sin x;
\]

\[
\phi_y = \cos ye^x + 2\cos x.
\]
Comment: 3 points off if one did not get the correct system.

Take the anti-derivative on both sides of the 2nd equation in the above system,

\[ \phi = \sin ye^x + 2 \cos xy + C(x). \]  

(1)

Comment: 2 points off if one get the wrong anti-derivative, especially if one did not put extra term \( C(x) \) in.

Take the partial derivative of (1) with respect to \( x \), then we have

\[ \phi_x = \sin ye^x - 2 \sin xy + C'(x). \]  

(2)

Comment: 1 point off if \( \phi_x \) is incorrect.

Comparing (2) with the \( \phi_x \) in the above system leads

\[ C'(x) = 0 \Rightarrow C(x) = A. \]

Specially set \( A = 0 \), then \( C(x) = 0 \).

Comment: 1 point off if one did not find \( C(x) \). It is still OK even one sets \( C(x) = A \) without specifying \( A = 0 \).

We get

\[ \phi = \sin ye^x - 2 \sin xy. \]

and then

\[ \phi = \sin ye^x - 2 \sin xy \equiv \text{Constant} \]

is an implicit solution of the modified equation.

Comment: One gets the correct form of \( \phi \), 1 point; setting \( \phi = \text{Constant} \), 2 points.

If one solves the above PDE system by starting from the first equation \( \phi_x \), and still finally gets \( \phi \), he/she can still get the corresponding points.