The Y A_k Ψ representation

This converts the stoichiometric representation dx/dt = S v(x) of a chemical
reaction network to the form
S v(x) = Y A_k Ψ(x) = Y G K Ψ(x).

Here A_k is the Laplacian of the complexes graph and Y is the complexes to species matrix. The factoring A_k = G K is found in
V. Katsnelson's UCSD undergrad honors thesis.
It also computes properties of A_k and the deficiency of the network.

```
(* This notebook presents all the commands available in the
 chemYAK.m file. *)
```

```
<< "Desktop/chemYAk.m"
```

```
chemYAk is loading...
```

```
chemYAk has loaded
```

```
S = Transpose[{{-1, 1, 0, 0, 1, 1}, {-1, 1, 0, 0, 0, 0}, {-1, 0, 1, 1, 0, 0}, {-1, 1, -1, 1, 0, 0},
    {0, 0, -1, 1, -1, 0}, {1, -1, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0}}](*stoichiometric matrix*);
S //
 MatrixForm
```

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(* matrixY takes the Stoichiometric matrix as intput and yields a
list of three matrices.
The first is matrix Y, the second is a list of the input complexes for all
the reactions (with repetitions), and the third is a list of the output
complexes for all the reactions (with repetitions). *)
```

```
Y = matrixY[S];
```

```
(* Here is the matrix Y. *)
```

```
MatrixForm[Y[[1]]]
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(* Here are the input complexes. They may be identified with
monomials whose exponents are given by the rows of this matrix. *)
```

```
MatrixForm[Transpose[Y[[2]]]]
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(* Here are the output complexes. *)
```

```
MatrixForm[Transpose[Y[[3]]]]
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(* matrixG takes in matrix Y, the input complexes matrix,
and the output complexes matrix, and yields matrix G. *)
```

```
G = matrixG[Y];
G//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

■ (* Check to make sure YG=S *)

```
Print["YG = ", MatrixForm[Y[[1]].G], ",  S=" , MatrixForm[S]];
Y[[1]].G == S
```

YG =
$$\begin{pmatrix} -1 & -1 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} -1 & -1 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
True
```

```
Psi = makeMonomial[-Y[[1]], x];
Print["Ψ(x)= ", MatrixForm[Psi]]
```

$$\Psi(x) = \begin{pmatrix} x[2]\,x[5]\,x[6] \\ x[1] \\ x[2] \\ x[3]\,x[4] \\ x[2]\,x[4] \\ x[1]\,x[3] \\ x[4] \\ x[3]\,x[5] \\ x[3] \end{pmatrix}$$

```
(* matrixK takes matrix G as input and yields matrix K. *)

K = matrixK[G];
K//MatrixForm
```

$$\begin{pmatrix} 0 & k[2,1] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,3] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,4] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k[6,5] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k[8,7] & 0 \\ 0 & 0 & k[3,2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k[7,9] & 0 & 0 \end{pmatrix}$$

```
Ak = G.K;
Ak // MatrixForm
```

$$\begin{pmatrix} 0 & k[2,1] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k[2,1]-k[2,3]-k[2,4] & k[3,2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,3] & -k[3,2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,4] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k[6,5] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k[6,5] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k[7,9] & k[8,7] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k[8,7] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k[7,9] & 0 & 0 \end{pmatrix}$$

■ ** We finally obtain our desired formula **

```
Print["YGKΨ(x)= ", MatrixForm[Y[[1]]], MatrixForm[G], MatrixForm[K], MatrixForm[Psi]]
```

$$
\text{YGK}\Psi(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
\begin{pmatrix} 0 & k[2,1] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,3] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k[2,4] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k[6,5] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k[8,7] & 0 \\ 0 & 0 & k[3,2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k[7,9] & 0 & 0 \end{pmatrix}
\begin{pmatrix} x[2]\,x[5]\,x[6] \\ x[1] \\ x[2] \\ x[3]\,x[4] \\ x[2]\,x[4] \\ x[1]\,x[3] \\ x[4] \\ x[3]\,x[5] \\ x[3] \end{pmatrix}
$$

```
(* The desired decomposition Y Ak Ψ(x). *)

decomposition[S]
```

```
{{{0, 1, 0, 0, 0, 1, 0, 0, 0}, {1, 0, 1, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 1, 0, 1, 1},
  {0, 0, 0, 1, 1, 0, 1, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 1, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, k[2, 1], 0, 0, 0, 0, 0, 0, 0}, {0, -k[2, 1] - k[2, 3] - k[2, 4], k[3, 2], 0, 0, 0, 0, 0, 0},
  {0, k[2, 3], -k[3, 2], 0, 0, 0, 0, 0, 0},
  {0, k[2, 4], 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, k[6, 5], 0, 0, 0},
  {0, 0, 0, 0, 0, -k[6, 5], 0, 0, 0}, {0, 0, 0, 0, 0, 0, -k[7, 9], k[8, 7], 0},
  {0, 0, 0, 0, 0, 0, 0, -k[8, 7], 0}, {0, 0, 0, 0, 0, 0, k[7, 9], 0, 0}},
 {x[2] x[5] x[6], x[1], x[2], x[3] x[4], x[2] x[4], x[1] x[3], x[4], x[3] x[5], x[3]}}
```

```
(* linkageClasses takes the Laplacian of a graph (e.g. the A_k matrix in the Y A_k Ψ(x))
 as an input. It returns a vector with two components. The first
 component gives a list of all the linkage classes. For example,
{1,3,4} would be a linkage class where the first,
third and fourth complex participate in reactions with each other. By definition,
the linkage classes are disjoint.

   The second component in our output is a list of 3-
 tuples for each different complex in our chemical network. A 3-tuple will list
  all the reactions a certain complex particpates in. The first vector in the 3-
 tuple gives the index i of a particular complex y_i. The second vector in the 3-
 tuple gives indices for of each complex that y_i particates in a reaction with,
and where y_i is an input for that reaction.
   The third vector in the 3-
 tuple gives indices for of each complex that y_i particates in a reaction with,
and where y_i is an output for that reaction. For example,
the 3-tuple {{5},{1,2},{2,4,6}} means we have the reactions y_5→y_1,
y_5→y_2, y_2→y_5, y_4→y_5, and y_6→y_2. *)

components = linkageClasses[Ak]

{{{1, 2, 3, 4}, {5, 6}, {7, 8, 9}},
 {{{1}, {}, {2}}, {{2}, {1, 3, 4}, {3}}, {{3}, {2}, {2}}, {{4}, {}, {2}},
  {{5}, {}, {6}}, {{6}, {5}, {}}, {{7}, {9}, {8}}, {{8}, {7}, {}}, {{9}, {}, {7}}}}

Print["The number of linkage classes is ", Length[components[[1]]]]

The number of linkage classes is 3

(*This gives the topological deficiency of a chemical reaction
 network. The input is the stoichiometric matrix. *)

topDeficiency[S]

1

(* The next formula gives the chemical deficiency of a chemical reaction
 network. Its input is the stoichiometric matrix. *)

matDeficiency[S]

1

(* Another example done from scratch: t.brucei. *)

S = {{0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {-1, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, -1, 1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, -1, 1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0,
    -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0, -2, 2, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0,
```

```
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 1, -1, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0, 1, -1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, -2, 2}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1,
  1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
  -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0},
 {1, -1, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, 1, 0, 0, 0, 0, 0, 0}};
{Y, Ak, Psi} = decomposition[S];
Y // MatrixForm
Ak // MatrixForm
Psi // MatrixForm
Print["The topological deficiency is ", topDeficiency[S]];
Print["The matrix deficiency is ", matDeficiency[S]];
```

0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

-k[1, 2] - k[1, 3]	k[2, 1]	k[3, 1]	0	0	0	0	0	0	0	0
k[1, 2]	-k[2, 1]	0	0	0	0	0	0	0	0	0
k[1, 3]	0	-k[3, 1]	0	0	0	0	0	0	0	0
0	0	0	0	k[5, 4]	0	0	0	0	0	0
0	0	0	0	-k[5, 4]	0	0	0	0	0	0
0	0	0	0	0	-k[6, 7]	k[7, 6]	0	0	0	0
0	0	0	0	0	k[6, 7]	-k[7, 6]	0	0	0	0
0	0	0	0	0	0	0	0	k[9, 8]	0	0
0	0	0	0	0	0	0	0	-k[9, 8]	0	0
0	0	0	0	0	0	0	0	0	-k[10, 11]	k
0	0	0	0	0	0	0	0	0	k[10, 11]	-
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$$\begin{pmatrix} x[38] \\ x[2] \\ x[1] \\ x[3]\,x[7] \\ x[2]\,x[6] \\ x[4] \\ x[3] \\ x[5]\,x[7] \\ x[4]\,x[6] \\ x[9]\,x[10] \\ x[5] \\ x[10] \\ x[9] \\ x[10]\,x[24] \\ x[12]\,x[25] \\ x[6]\,x[14] \\ x[7]\,x[12] \\ x[32] \\ x[14] \\ x[34] \\ x[15] \\ x[16]\,x[17] \\ x[15]\,x[18] \\ x[27] \\ x[16] \\ x[11]\,x[24] \\ x[9]\,x[25] \\ x[9]\,x[21] \\ x[11]\,x[20] \\ x[20]\,x[22] \\ \frac{1}{2}\,x[21]\,x[23] \\ x[6]\,x[13] \\ x[7]\,x[11] \\ x[39] \\ x[13] \\ x[26] \\ x[18]\,x[37] \\ x[17] \\ x[6]\,x[8] \\ x[7]^2 \\ x[17]\,x[19] \\ x[18]^2 \end{pmatrix}$$

The topological deficiency is 0

The matrix deficiency is 0

The linkage classes:

```
{{{1, 2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {12, 13}, {14, 15}, {16, 17}, {18, 19, 20, 21}, {22, 23},
  {24, 25}, {26, 27}, {28, 29}, {30, 31}, {32, 33}, {34, 35, 36}, {37, 38}, {39, 40}, {41, 42}},
 {{{1}, {2, 3}, {2, 3}}, {{2}, {1}, {1}}, {{3}, {1}, {1}}, {{4}, {}, {5}}, {{5}, {4}, {}},
  {{6}, {7}, {7}}, {{7}, {6}, {6}}, {{8}, {}, {9}}, {{9}, {8}, {}}, {{10}, {11}, {11}},
  {{11}, {10}, {10}}, {{12}, {13}, {13}}, {{13}, {12}, {12}}, {{14}, {15}, {15}},
  {{15}, {14}, {14}}, {{16}, {17}, {17}}, {{17}, {16}, {16}}, {{18}, {19, 20}, {19, 20}},
  {{19}, {18}, {18}}, {{20}, {18, 21}, {18, 21}}, {{21}, {20}, {20}}, {{22}, {}, {23}},
  {{23}, {22}, {}}, {{24}, {}, {25}}, {{25}, {24}, {}}, {{26}, {27}, {27}},
  {{27}, {26}, {26}}, {{28}, {29}, {29}}, {{29}, {28}, {28}}, {{30}, {31}, {31}},
  {{31}, {30}, {30}}, {{32}, {33}, {33}}, {{33}, {32}, {32}}, {{34}, {35, 36}, {35, 36}},
  {{35}, {34}, {34}}, {{36}, {34}, {34}}, {{37}, {38}, {38}}, {{38}, {37}, {37}},
  {{39}, {40}, {40}}, {{40}, {39}, {39}}, {{41}, {42}, {42}}, {{42}, {41}, {41}}}}
```

```
(* A (too) big example *)

S = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 1, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {-1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, -1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{1, 0, 0, 0, 2, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, -1,
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0, 0, -1, 0, 0, -1, 0, 0, 0, 0, -1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
-1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0}};
{Y, Ak, Psi} = decomposition[S];
Y // MatrixForm
Ak // MatrixForm
Psi // MatrixForm
Print["The topological deficiency is ", topDeficiency[S]];
Print["The matrix deficiency is ", matDeficiency[S]];
Print["The linkage classes: ", linkageClasses[Ak]];
```

```
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  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   0 0 0 1 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 0 0 0
  0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 0 0 0
```

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-k[1, 2]	k[2, 1]	0	0	0	0	0	0	0
k[1, 2]	-k[2, 1]	0	0	0	0	0	0	0
0	0	-k[3, 4] - k[3, 66]	k[4, 3]	0	0	0	0	0
0	0	k[3, 4]	-k[4, 3]	0	0	0	0	0
0	0	0	0	-k[5, 6]	k[6, 5]	0	0	0
0	0	0	0	k[5, 6]	-k[6, 5]	0	0	0
0	0	0	0	0	0	-k[7, 8]	k[8, 7]	0
0	0	0	0	0	0	k[7, 8]	-k[8, 7]	0
0	0	0	0	0	0	0	0	-k[9, 10
0	0	0	0	0	0	0	0	k[9, 10]
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0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
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0	0	k[3, 66]	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

x[9] x[10]
x[6] x[14]
x[43]
x[15]
x[6] x[41]
x[7] x[42]
2 x[18] x[21] x[47]
4 x[8] x[41] x[48]
$x[10]^2$
x[13] x[14]
x[16] x[48] x[68]
x[11] x[18] x[47]
x[11] x[41]
x[12] x[42]
x[10] x[41] x[57]
x[14] x[39]
3 x[14] x[39] x[41]

```
4 x[10] x[42] x[57]
x[16]
x[17]
x[15] x[18] x[41]
x[8] x[39] x[55]
2 x[39] x[42] x[61]
0.5 x[41]^2 x[53] x[62]
x[41] x[44]
x[42] x[45]
x[39] x[56]
x[2]
x[21] x[41]
x[22] x[42]
x[19] x[31]
x[26]
x[23] x[57]
x[26] x[39]
x[27]
x[28]
x[24] x[66]
x[25] x[29]
x[46]
x[29] x[39]
2 x[29] x[41]
2 x[30] x[42]
x[5] x[41] x[50]
x[32] x[49]
x[1] x[41] x[48]
x[31] x[47] x[57]
x[32] x[59]
x[33] x[56]
x[10] x[34] x[41] x[57]
x[14] x[36] x[51]
x[14] x[35] x[39]
x[11] x[41] x[50] x[51]
x[36] x[39] x[49]
x[36] x[51]
x[34] x[39]
x[36]^2 x[49]
x[11] x[34] x[41] x[50]
x[36] x[41]
x[37] x[42]
x[16] x[50] x[64]
x[4] x[49]
x[39]
x[40]
x[11] x[16] x[50]
x[43] x[49]
x[38] x[66]
x[41] x[48] x[59]
x[44] x[47]
x[18] x[41] x[46]
x[8] x[38] x[39]
```

x[41] x[48] x[55]
x[46] x[47]
x[16] x[48] x[59]
x[16] x[50] x[59]
x[46] x[49]
x[42]^2 x[47] x[62]
x[41]^3 x[48] x[61]
x[48] x[49]
x[47] x[50]
x[51]
x[52]
x[53]
x[54]
x[8] x[16] x[48]
x[18] x[47] x[59]
x[10] x[26] x[41]
x[14] x[23]
x[8] x[27]
x[18] x[59]
x[23]
x[32]
x[1] x[10]
x[3] x[14]
x[4] x[41]
x[5] x[39]
x[3]
x[41] x[57]
x[42] x[58]
x[41] x[55] x[57]
x[16] x[39] x[56]
x[10] x[16] x[56]
x[14] x[55]
2 x[13] x[41] x[56] x[57]
x[14] x[39] x[59]
x[9] x[18]
x[8] x[57]
x[14] x[59]
x[10] x[41] x[56]
x[41] x[59]
x[42] x[60]
x[69]
x[64]
x[63]
x[41]^2 x[66]
x[42]^2 x[67]
x[41] x[67]
x[42] x[66]
x[24] x[62]
x[25] x[61]
x[10] x[57] x[68]
x[14] x[18] x[66]
x[20] x[23]

$$\begin{pmatrix} x[31]\, x[65] \\ x[41]^2\, x[47]\, x[50] \\ x[42]^2\, x[48]\, x[49] \\ x[63]\, x[69] \\ x[23]\, x[31] \\ x[20]\, x[69] \\ x[31] \\ x[19] \end{pmatrix}$$

The topological deficiency is 11