- Inner product: generalization of dot product on $\mathbb{R}^n$. (Heuristic/intuition: testing in a certain direction).
- Examples: $\mathbb{R}^n$; $\mathbb{C}^n$; standard inner product (dot product); weighted inner products; continuous functions on an interval with integration; matrix adjoint and Frobenius inner product.
- Inner product space; choice of inner product determines geometry.
- Continuous functions into the complex numbers; inner products for such functions.
- Properties of the inner product: sesquilinear; faithful.
- Inner product induces a notion of length (or norm).
- Properties of the norm: scaling; faithful; Cauchy-Schwarz.
- Applications of Cauchy-Schwarz inequality: triangle inequality; specific inequalities in $\mathbb{R}^n$ (example: \( \frac{(a_1 + \cdots + a_n)^2}{n} \leq a_1^2 + \cdots + a_n^2 \)); in $M_{n\times n}(\mathbb{R})$ (example: let $A_n$ be symmetric and positive definite, then \( a_{ij}^2 \leq a_{ii}a_{jj} \)); and integral inequalities (example: \( (\int_0^1 f(x) \, dx)^2 \leq \int_0^1 f(x)^2 \, dx \)).
- Orthogonal vectors; orthogonal subsets; unit vectors; orthonormal vectors; normalizing an orthogonal set; Pythagorean theorem; example of \( f(x) = 1 \) and \( g(x) = x \) in $C([-1,1])$.
- Example of orthogonal functions $f_n(t) = e^{int}$ (look up Fourier transform if interested in motivation https://www.youtube.com/watch?v=spUNpyF58BY).
- Orthonormal basis; basis representation with respect to an orthogonal (orthonormal) basis; orthogonal sets that don’t contain the zero vector are linearly independent.