SO AS TO NOT DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE.

ANSWERS TO THE TRUE/FALSE QUESTIONS DO NOT NEED TO BE JUSTIFIED; HOWEVER, INCORRECT ANSWERS TO THE TRUE/FALSE QUESTIONS ARE PENALIZED. IN PARTICULAR, A CORRECT ANSWER IS WORTH 5 POINTS, AN INCORRECT ANSWER IS WORTH -5 POINTS, AND A BLANK ANSWER IS WORTH 0 POINTS.

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

THIS EXAM WILL BE SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED.

THE EXAM CONSISTS OF 5 TRUE/FALSE QUESTIONS AND 3 LONGER FORMAT QUESTIONS. YOUR ANSWERS TO THE LONGER FORMAT QUESTIONS SHOULD BE CAREFULLY JUSTIFIED. YOU ARE ALLOWED TO USE RESULTS FROM THE TEXTBOOK, HOMEWORK, AND LECTURE.
1. (25 points) Label the following statements as true or false. Any ambiguous answer (for example, resembling a hybrid of T and F) will be treated as an incorrect answer.

(a) ________ If $X \sim \text{Exp}(\lambda)$ and $c > 0$, then $cX \sim \text{Exp}(c\lambda)$.

(b) ________ If $X \sim \mathcal{N}(0, \sigma^2)$ and $Y \sim \mathcal{N}(0, \rho^2)$ are such that $\sigma^2 > \rho^2$, then

$$\mathbb{P}(|X| < r) > \mathbb{P}(|Y| < r)$$

for every $r > 0$.

(c) ________ If the CDF $F_X$ of a random variable $X$ is differentiable at all but finitely many points, then $X$ is necessarily a continuous random variable with density $f_X = F'_X$ at the points where $F_X$ is differentiable.

(d) ________ If $X$ and $Y$ are independent continuous random variables with densities $f_X(r)$ and $f_Y(r)$ respectively and CDFs $F_X(r)$ and $F_Y(r)$ respectively, then the random variable

$$Z = \max(X, Y) = \begin{cases} X & \text{if } X \geq Y \\ Y & \text{if } X < Y \end{cases}$$

is necessarily continuous with density $f_Z(r) = f_X(r)F_X(r) + f_Y(r)F_Y(r)$.

(e) ________ If $X$ and $Y$ are continuous random variables (i.e., there exist density functions $f_X, f_Y : \mathbb{R} \to \mathbb{R}_{\geq 0}$ respectively), then $X$ and $Y$ are necessarily jointly continuous (i.e., there exists a joint density function $f_{(X,Y)} : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$).
2. (25 points) Suppose that the time it takes for you to complete your probability homework is distributed according to an exponential random variable with the rate 1 full homework assignment per hour. You start your homework at 8:00 PM. Your bedtime is 10:00 PM. If you finish your homework before your bedtime, you watch TV until your bedtime and then go to sleep. If you do not finish by your bedtime, you go to sleep anyway, and so you do not watch TV at all. Let $Y$ be the random variable that measures the amount of time that you spend watching TV.

(a) (15 pts) Calculate the CDF of $Y$. You may choose to write $Y$ in terms of hours or in terms of minutes.
(b) (10 pts) Calculate the expected value $\mathbb{E}[Y]$. Again, you may choose to write $Y$ in terms of hours or in terms of minutes.
3. (25 pts) Let $T$ be the triangle in $\mathbb{R}^2$ with vertices $(0, 0), (1, 0), (1, 1)$. Let $P = (X, Y)$ be a uniform random point in the triangle $T$.

(a) (15 pts) What is the joint density $f_{(X,Y)}(x, y)$ of $P$? What is the marginal density $f_X(x)$ of $X$? What is the marginal density $f_Y(y)$ of $Y$? Are $X$ and $Y$ independent?
(b) (10 pts) Let $Z = Y - X$. Determine the CDF of $Z$. Compute the expectation $\mathbb{E}[Z]$. 
4. (25 pts) Let $X_1$, $X_2$, and $X_3$ be independent random variables such that

$$X_1 \sim \text{Unif}([0, 1]), \quad X_2 \sim \text{Unif}([0, 1]), \quad \text{and} \quad X_3 \sim \text{Unif}([0, 1]).$$

Let $Y$ denote the 2nd largest value among the 3 numbers $X_1, X_2, X_3$. For example, if $X_1 = \frac{1}{4}$, $X_2 = \frac{2}{3}$, and $X_3 = 1$, then $Y = \frac{2}{3}$. As another example, if $X_1 = \frac{1}{4}$, $X_2 = \frac{1}{4}$, and $X_3 = 1$, then $Y = \frac{1}{4}$. Determine the CDF $F_Y$ and density $f_Y$ of $Y$. 
(ADDITIONAL SPACE FOR WORK, CLEARLY INDICATE THE PROBLEM YOU ARE WORKING ON)
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