From Graph Theory to Yang-Mills Theory via Math 202B

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Abstract

There are many interesting matrices associated to graphs. We all know about the adjacency matrix and the Laplacian, the basic matrices of spectral graph theory. The distance matrix is another interesting one - it was famously shown by Graham and Pollack that distance determinants of trees depend only on the number of vertices. The characteristic polynomial of distance matrices of trees was further studied by Graham and Lovasz, who found many interesting properties. Recently, graph theorists have begun to consider "exponential distance matrices" of graphs, obtained by taking the entrywise exponential of the usual distance matrix, and have proved some basic theorems on their eigenvalues for simple families of graphs. Taking a less myopic view of the mathematical landscape quickly reveals that exponential distance matrices appeared some thirty years ago in quantum physics, when Zagier explicitly evaluated the determinant of the exponential distance matrix of the Coxeter-Cayley graph of the symmetric group as the main step in proving the existence of a Hilbert space representation of deformed commutation relations interpolating between bosons and fermions. I will describe parallel results for the Hurwitz-Cayley graph of the symmetric group and explain their relation to gauge-string/dualities in Yang-Mills theory. As in Zagier's study, the main tools come from discrete harmonic analysis, aka the character theory of finite groups, and some basic aspects of symmetric function theory also play an important role. From a pedagogical perspective, the moral of the story is that it's good to imbibe some algebra with your combinatorics, and plain old matrices just don't cut it.