

Combinatorial characterizations of smooth positroid varieties via pattern avoidance, spirographs, and Johnson graphs

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Abstract

Positroids are certain representable matroids originally studied by Postnikov in connection with the totally nonnegative Grassmannian and now used widely in algebraic combinatorics. The positroids give rise to determinantal equations defining positroid varieties as subvarieties of the Grassmannian variety. Rietsch, Knutson-Lam-Speyer, and Pawlowski studied geometric and cohomological properties of these varieties. In this talk, we continue the study of the geometric properties of positroid varieties by establishing several equivalent conditions characterizing smooth positroid varieties using a variation of pattern avoidance defined on decorated permutations, which are in bijection with positroids. This allows us to give several formulas for counting the number of smooth positroids according to natural statistics on decorated permutations. Furthermore, we give a combinatorial method for determining the dimension of the tangent space of a positroid variety at the torus fixed points using an induced subgraph of the Johnson graph. We will conclude with some open problems in this area.

This talk is based on joint work with Jordan Weaver and Christian Krattenthaler.