

Weighted b -Hurwitz numbers from W -algebras

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Abstract

Weighted Hurwitz numbers were introduced by Harnad and Guay-Paquet as objects covering a wide class of Hurwitz numbers of various types. A particularly strong property of Hurwitz numbers is that they are governed by the celebrated topological recursion (TR) of Chekhov–Eynard–Orantin: a universal algorithm that allows computation of them recursively with respect to their topology. The program of understanding how TR can be used to compute different types of Hurwitz numbers was carried out over the last two decades by considering each case separately, and finally, the general case of rationally-weighted Hurwitz numbers was recently proved by Bychkov–Dunin-Barkowski–Kazarian–Shadrin.

We will discuss a more general case of weighted b -Hurwitz numbers that arise naturally in the context of symmetric functions theory and matrix models. We show that their generating function satisfies the so-called W -constraints - certain explicit differential equations arising from representations of W -algebras. We will focus on a transition from an algebraic/geometric background to a combinatorial one, which turned out to be crucial in our work. Our result gives a new explanation of the remarkable enumerative properties of Hurwitz numbers following from TR, and extends it to the b -deformed case. This is joint work with Nitin Chidambaram and Kento Osuga.