Duality for polynomials

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Abstract

In recent years, many mathematicians have contributed to a combinatorial theory for the polynomial ring $\mathbb{C}[x_1, x_2, ...]$ similar to symmetric function theory. Beginning with Schubert polynomials and later key polynomials, numerous bases have been introduced whose monomials have combinatorial interpretations. In the theory of harmonic polynomials, there is a natural inner product for the polynomial ring with monomials as an orthogonal basis. Duality with respect to this inner product is characterized by a Cauchy type identity. We show how to interpret this duality combinatorially. As a byproduct, we recover Postnikov and Stanley's dual Schubert polynomials and introduce a novel family of dual key polynomials whose further properties remain uninvestigated.