The Kostka semigroup and its Hilbert basis

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Abstract

The Kostka semigroup consists of pairs of partitions with at most r parts that have positive Kostka coefficient. For this semigroup, Hilbert basis membership is an NP-complete problem. We introduce KGR graphs and conservative subtrees, through the Gale-Ryser theorem on contingency tables, as a criterion for membership. In our main application, we show that if a partition pair is in the Hilbert basis then the partitions are at most r wide. We also classify the extremal rays of the associated polyhedral cone; these rays correspond to a (strict) subset of the Hilbert basis. In an appendix, the second and third authors show that a natural extension of our main result on the Kostka semigroup cannot be extended to the Littlewood-Richardson semigroup. This furthermore gives a counterexample to a recent speculation of P. Belkale concerning the semigroup controlling nonvanishing conformal blocks.