Clique and Berge Supersaturation for $K_{2,t}$

Sam Spiro
Rutgers University

Abstract

A famous conjecture of Erdős and Simonovits says that if G is an n vertex graph with much more than (n, F) edges, then G contains about as many copies of F as the random graph of the same density. In this talk we show that several natural generalizations of this conjecture fails to be true. In particular, we show that for large t, there exist n vertex graphs with $\Theta(kn^{3/2})$ triangles such that G contains a total of $k^t n^{3/2+o(1)}$ copies of $K_{2,t}$ (with the random graph of the same triangle density containing $\Theta(k^{2t/3}n^2)$ copies), and we show that this bound is essentially best possible for $k \leq n^{1/2t}$. Our constructions rely on solving certain unbalanced bipartite Turán problems using random polynomial graphs. This is joint work with Quentin Dubroff, Benjamin Gunby, and Bhargav Narayanan.