The Kronecker coefficients from Jacobi’s alternant formula for the Schur function

Sheila Sundaram
Pierrepont School, Westport, CT

Abstract

The Kronecker coefficients are the notoriously elusive structure constants for the decomposition into irreducibles of the tensor product of irreducible representations of the symmetric group. In recent joint work with Marni Mishna and Mercedes Rosas, we study the piecewise quasipolynomial nature of the Kronecker function using tools from polyhedral geometry. In this talk I will describe this approach, which begins with Jacobi’s definition of the Schur function as a quotient of alternants, an idea originally exploited by Rosas in her thesis. I will then illustrate its power by focusing on the first nontrivial case, showing how we derive, for this case, new exact formulas and an upper bound for the Kronecker coefficients (the atomic Kronecker coefficient) as well as other properties. The polyhedral geometry gives a curious digraph whose nodes are certain monomials in the alternant. An additional advantage of this approach is that asymptotic estimates for dilations can be computed using techniques of analytic combinatorics in several variables.