On elements of $a$-value 2 in Coxeter groups

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Abstract

The $a$-function on a Coxeter group $W$ is a function $a : W \to \mathbb{N}$ defined by Lusztig which is intimately related to the partition of $W$ into Kazhdan–Lusztig cells and to the representation theory of the Hecke algebra of $W$. It is known that the identity element of $W$ is the only element with $a$-value 0, while a non-identity element has $a$-value 1 if and only if it has a unique reduced word. However, as its definition relies on the Kazhdan–Lusztig basis of the Hecke algebra, the $a$-function is often difficult to compute for general elements.

In this talk we will focus on elements of $a$-value 2, or $a$-2 elements. We show that $a$-2 elements are fully commutative in the sense of Stembridge, which allows us to associate to them certain posets called heaps and, in many cases, certain generalized Temperley–Lieb diagrams. Using heaps and Temperley–Lieb diagrams, we conjecture a combinatorial characterization of $a$-2 elements, classify all Coxeter groups with finitely many $a$-2 elements, and enumerate $a$-2 elements for all groups from the classification. Joint with Richard Green.