

New bounds on piercing numbers and line-piercing numbers in families of convex sets in the plane.

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Abstract

A family F of sets has the (p, q) property if among any p members of it some q intersect. F has the $T(k)$ property if every k sets in F are intersected by a line. We prove that if F is a family of convex sets in the plane with the $(p + 1, 2)$ property then there are $\lfloor (p/2) + 1 \rfloor$ lines whose union intersects all the sets in F , and this bound is tight. We use this result to prove new bounds on the piercing numbers in families of convex sets in the plane with the $(p, 2)$ property, in terms of the matching numbers of their pairwise intersection families. We further prove a conjecture of Eckhoff from 1993, asserting that if a family of convex sets in the plane has the $T(3)$ property then there are 3 lines whose union intersects all the sets in it. Rainbow versions of these results are also proved. The proofs use the topological KKM theorem and its colorful generalization.