

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C \neq (A \times B) \times C,$$

Lagrange' identity:  $(v \times w) \cdot (a \times b) = (v \cdot a)(w \cdot b) - (v \cdot b)(w \cdot a)$

$$\frac{ds}{dt} = |\alpha'(t)| = \nu(t), \quad T(t) := \frac{\alpha'(t)}{\nu(t)}, \quad \kappa(s) := \left| \frac{dT}{ds} \right|, \quad N(s) := \frac{\frac{dT}{ds}}{\kappa(s)}, \quad B := T \times N, \quad \tau(s) := \frac{dN}{ds} \cdot B$$

$$\begin{aligned} \frac{dT}{ds} &= \kappa N \\ \frac{dN}{ds} &= -\kappa T + \tau B \\ \frac{dB}{ds} &= -\tau N. \end{aligned}$$

$$\begin{aligned} \frac{dT}{dt} &= \frac{dT}{ds} \frac{ds}{dt} = \kappa \frac{ds}{dt} N \\ \frac{dN}{dt} &= \frac{dN}{ds} \frac{ds}{dt} = -\kappa \frac{ds}{dt} T + \tau \frac{ds}{dt} B \\ \frac{dB}{dt} &= \frac{dB}{ds} \frac{ds}{dt} = -\tau \frac{ds}{dt} N. \end{aligned}$$

$$\kappa = \frac{|\alpha' \times \alpha''|}{\nu^3}, \quad B = \frac{\alpha' \times \alpha''}{|\alpha' \times \alpha''|}, \quad \tau = \frac{(\alpha' \times \alpha'') \cdot \alpha'''}{\kappa^2 \nu^6} = \frac{(\alpha' \times \alpha'') \cdot \alpha'''}{|\alpha' \times \alpha'''|^2}.$$

The involute:  $\mathcal{I}(t) = \alpha(t) - s(t) \frac{\alpha'(t)}{|\alpha'(t)|} = \alpha(t) - s(t)T(t)$ , the evolute:  $\mathcal{E}(t) = \alpha(t) + \frac{N(t)}{\kappa(t)}$ .

Given  $\kappa(s)$ , put  $\theta(s) := \int_{s_0}^s \kappa(u) du$ . A plane curve with curvature  $\kappa$  is given by

$$\beta(s) = \left( \int_{s_0}^s \cos(\theta(u)) du, \int_{s_0}^s \sin(\theta(u)) du \right).$$

$$U := \frac{\varphi_u \times \varphi_v}{|\varphi_u \times \varphi_v|}, \quad S_P(V) = -\nabla_V U.$$

The patch for a surface of revolution obtained by rotating  $\alpha(u) = (g(u), h(u))$  around the  $x$ -axis:

$$\varphi(u, v) = (g(u), h(u) \cos v, h(u) \sin v), \quad K = \frac{g'(g''h' - g'h'')}{h(g'^2 + h'^2)^2}, \quad H = \frac{h(g''h' - g'h'') + g'(g'^2 + h'^2)}{2h(g'^2 + h'^2)^{3/2}},$$

when  $\alpha$  has unit speed,  $K = \frac{-h''}{h}$ .

The normal curvature in the direction of a unit vector  $u$ :  $k(u) := S_p(u) \cdot u$

Euler's formula:  $k(u) = k_1 \cos^2 \theta + k_2 \sin^2 \theta$

$$\begin{aligned} S(\varphi_u) \cdot \varphi_u &= \varphi_{uu} \cdot U, \quad S(\varphi_u) \cdot \varphi_v = S(\varphi_v) \cdot \varphi_u = \varphi_{uv} \cdot U, \quad S(\varphi_v) \cdot \varphi_v = \varphi_{vv} \cdot U \\ K &= \frac{ln - m^2}{EG - F^2}, \quad H = \frac{Gl + En - 2Fm}{2(EG - F^2)}, \quad k_1 = H + \sqrt{H^2 - K}, \quad k_2 = H - \sqrt{H^2 - K} \end{aligned}$$

$$\text{When } F = 0, \quad K = -\frac{1}{2\sqrt{EG}} \left( \frac{\partial}{\partial v} \left( \frac{E_v}{\sqrt{EG}} \right) + \frac{\partial}{\partial u} \left( \frac{G_u}{\sqrt{EG}} \right) \right)$$

$$\alpha'' = (\alpha'' \cdot T) T + (\alpha'' \cdot (U \times T)) (U \times T) + (\alpha'' \cdot U) U$$

$$\kappa_\alpha^2 = k_n(T_\alpha)^2 + (k_g)_\alpha^2$$