MATH 150B

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April 21 2017: Practice problems for first midterm

Please explain or prove all your assertions and show your work. Please state all the definitions, propositions, theorems, lemmas that you use precisely.

Please make sure to review all definitions, statements of theorems, proofs done in class, practice problems and homework problems.

There will be five questions on the midterm: 1 about definitions and statements of theorems, 1 proof, and 3 problems.

Although homework 3 is due after the test, its material is covered by the test: please make sure to do homework 3 before the midterm.

Good luck!

(1) A ruled surface $\varphi(u,v) = \beta(u) + v\delta(u)$ is developable if its unit normal U is constant along the rulings, i.e., $U_v = 0$. Show that a ruled surface is developable if and only if its Gauss curvature is 0.

(2) Without using the previous exercise show that cones and cylinders are developable.

(3) Let M be a surface and p and q two points on M. Let α be a piecewise regular curve on M from p to q. Prove that if α has the shortest length of all curves from p to q on M, then α is a geodesic.

    Hint: One can define a distance on M by letting the distance between two points be the length of the shortest curve joining them. To show that α has no corners, use the following result: given $p \in M$, there exists $\epsilon = \epsilon_p > 0$ such that every point q of distance ϵ from p has a normal neighborhood of radius ϵ (you can find a proof in DoCarmo's book).

(4) For a non-unit speed curve $\alpha(t)$ with speed $\nu(t)$, prove that

$$\alpha'' = \nu' T + \kappa_g \nu^2 U \times T + (\alpha'' \cdot U)U$$

    where κ_g is the geodesic curvature of α.