Prof. R. L. Graham
Bell Laboratories
Murray Hill
New Jersey 07979

JUL 17 1995

done 7/18/95
$1000

June 10, 1995

check # 3098

Dear Dr Graham,

I met with Prof. Erdős on the Hypergraphs and Symmetric Structures Conference which was held between 19 and 23 June, 1995 at Balatonlelle, Hungary.

Prof. Erdős told me that I can borrow 1000 dollars of his money if necessary. In September, 1995 I shall come to the US (Auburn, AL) to begin my graduate study. I arrive on 14 of September at Auburn, but unfortunately I shall receive my first check a month later. For this month long period I shall need this money. I will repay it as soon as I can.

Prof. Erdős has written a letter concerning this. I enclosed his letter.

TECHNION · ISRAEL INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS
32000 Haifa, Israel 1992 XII 16

הטכניון — מכון טכנולוגי לישראל
הפקולטה למתמטיקה
חיפה 32000

Dear Ron

Another nut left the now fast sinking ship. I will be here till the end of january.

Let $p_1, p_2 \dots p_k$ be a sequence of primes. $a_1 < a_2 < \dots$ is the sequence of integers composed of the p's. Is it true that all (or all large) integers are of the form $\sum a_i$, $a_i \nmid a_j$. I stupidly thought that this is difficult for 2, and 3 but Jansen and many others found the simple proof by induction. Lewin and I proved now that all $n > 31$ is of this form for 2, 5, 7, but perhaps it is not easy to find a good nec + suff condition for $p_1 \dots p_k$. Probably two primes suffice only for 2 and 3 but I may be completely wrong. Is it true that every $n > n_0$ can be written in the form

$$n = a_{i_1} + a_{i_2} + \dots + a_{i_r}, \quad a_{i_1} < a_{i_2} < \dots < a_{i_k} < 2a_{i_1}$$

$a_i = 2^\alpha 3^\beta$. I could not decide. Is this hard or do I overlook a trivial point.

I have the following very nice problem with Burr (in our paper dedicated to Rankins 70-th birthday, Glasgow Math Journal about 1985) Color the squares by 2 colors is it true that every $n > n_0$ is the sum of monochromatic distinct squares.?

Merry christmas + happy new year to you + Fam

au revoir E. P.

My new year of course can not be happy for obvious reasons.

Fax.: 972-4-324654 Bitnet: MATH111@Technion Telex: 46406 TECON IL Telephone: (04)294272, (04)223071 :טלפון

Dear Ron (1978 XI 3)

In the problem session Heath Brown asked: Is it true that the number of solutions of

$$\frac{1}{n_1}+\frac{1}{n_2}-\frac{1}{n_3}-\frac{1}{n_4} < \frac{1}{X^3}, \quad 0<n_i<X \quad (n_i \text{ integer})$$

is less than $X^{2+\varepsilon}$? Perhaps it is $< X^{2-\varepsilon}$ if one discards $n_1=n_3$, $n_2=n_4$ but I have not yet settled it.

He also mentioned the following nice question of Huxley: Is it true that

(1) $$\sum_{\substack{p_{n+1}-p_n > \frac{1}{3}\log x \\ p_n<x}} 1 > c\, x/\log x$$

(1) will of course hold with any α ($c=c(\alpha)$) instead of $\frac{1}{3}$ but I can not do it with $\frac{1}{3}$. $\sum_{\substack{p_{n+1}-p_n>\log x \\ p_n<x}} (p_{n+1}-p_n) > cx$ follows from Bruns method without too much difficulty.

I wrote some nonsense in my last letter. Trivially for large x $\pi(x)-\pi(x-y) > (1-\varepsilon)\pi(y)$ can not hold for all y. If $x-y = x^{7/12}$ then $\pi(x)-\pi(x-y) = (1+o(1))\frac{x^{7/12}}{\log x}$ the reasonable question is: Are there inf many values of ~~y~~ x for which for every $y = o(x^{\varepsilon})$

$\pi(x) - \pi(x-y) > (1-\varepsilon)\pi(y)$.

The answer is obviously no but here I do not see the proof. I just phoned Vera - all is well - there will be a meeting at Kalamazoo in May and I am invited to preach there. It seems I have to ~~wr~~ wire them since they would like to have the answer by Nov 5. Please send reprints of all our papers on binomial coefficients to Prof H.J. Kanold and H. Harborth both are at the university of Braunschweig.

Harborth R 79, 69, 65

H.J. Kanold 79, 69

Do you know examples of

$$\frac{u(u+1)\cdots(u+t)}{v(v+1)\cdots(v+t)} = \text{integer}$$

for $\frac{u}{2} < v < u$? ($v+t < u$) i.e. $\frac{456}{345}$ is not allowed.

I seem to remember that we once proved that for every i if $n > n_0(\varepsilon, i)$ then $\binom{n}{i} | \binom{n}{k}$ for all k except possible εn values of k, Harborth proved a somewhat weaker result namely that this holds on the average if $n < x$. But in this form we could not have proved it: if $n = 2^k$ $\binom{2^k}{2l} \not\equiv 0 \pmod{2^k}$, thus perhaps Harborths result is as good as can be expected. Now I remember I think we had that for every i there are integers n so that for some $k \le \frac{n}{2}$, $\binom{n}{i} | \binom{n}{k}$.

Please send a copy of our paper on Ramsey ~~ne~~ problems (Keszthely 1973) to: Dr Harborth Bienroder Weg 47 3300 Braunschweig Germany

Bienroder

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DEPARTMENT OF MATHEMATICS
32000 Haifa, Israel

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1992 XII 30

Dear Ron,

Are you still alive? I heard nothing from you for an infinitely long time. I am in Israel for 4 more weeks. If you can phone me on your free phone the phone number of my office is 972 4 294173.

Divide the plane into $\aleph_0$ parts. Must you have a polygon (convex polygon?) of rational area? Also can it happen that the area of the monochromatic triangles have measure 0? Perhaps all this follows easily from the construction of Kunen which you mention in your paper. Has it been published?

Can you split the squares into 2 classes such that there should be no monochromatic arithmetic progression also $x^2+y^2=z^2$ should have no monochromatic solution

I hope you will write your paper on Euclidean Ramsey for Israel
I enclose a cheque for 500 dollars, also please send 25 dollars to the Quakers.

Kind regards + happy new year to you + Fan au revoir
E.P.

Fax.: 972-4-324654 Bitnet: MATH111@Technion Telex: 46406 TECON IL Telephone: (04)294272, (04)223071 :טלפון

1993 III 3

Memphis State University

901/678-2482
Mathlib@memstvx1
MSU FAX# 901/678-3299

Dear Fan + Ron,

I may be in New Jersey before I fly to England on March 19. I will probably arrive on or around March 15.

Please pay the dues for Science but it is not necessary to send it to Hungary by air mail.

Did you send the subscription for the National Geographic for Ms SZÉKELY Virginia NAGYENYED u 7 Budapest Hungary? You did do this for the last few years.

Kind regards, au revoir

E. P.

Can you color the squares with two colors so that $x^2+y^2=2z^2$ should not be solvable monochromatically?

Let $a_1<a_2<\dots$ be an infinite sequence of integers $f(n)$ is the number of solutions of $a_i+a_j=n$. An old conjecture of Turán and myself states that if $f(n)>0$ for all $n>n_0$ then $\overline{\lim}\, f(n)=\infty$. Perhaps $\sum_{\substack{n<x \\ f(n)>0}} 1 = x+o(x)$ will suffice. Sárközy + I now thought that if $a_1<a_2<\dots$ and $b_1<b_2<\dots$ are two infinite sequences for which every n is of the form a_i+b_j and if $a_n/b_n \to 1$ then the number of solutions of $n=a_i+b_j$ is unbounded. $a_n/b_n \to 1$ can not entirely be neglected to see this let the a's be $\sum \varepsilon_i 2^{2i}$ and the b's $\sum \varepsilon_i 2^{2i+1}$, $\varepsilon_i=0$ or 1

1992 XI 16

Mathematatisches Forschungsinstitut

762 Oberwolfach-Walke,
Lorenzenhof
Telefon Wolfach 07834 / 311 und 512

NOV 2 1992

Dear Ron,

Today I go to Braunschweig and on the 19-th I fly to Budapest and a few days later I fly to Haifa (Technion Math Dept Haifa Israel)

A very old conjecture of mine stated that if $a_1 < a_2 < \dots < a_k \le n$ is such that you can not find $r+1$ a-s which are pairwise relatively prime then max k is obtained by taking the multiples of the first r primes. An Armenian Ma[thematician] named Levon Khachatrian who is in Bielefeld now found a counterexample. No doubt the conjecture remains true for fixed r and $n > n_0(r)$ probably $n > (1+\varepsilon) p_r^2$ will suffice. Here is his counterexample. Take a p_m for wh[ich] $p_m p_{m+9} > p_{m+8}^2$ it is easy to find such primes but may be hard to prove that there are inf many such m. Let $n = p_m p_{m+9}$

My sequence is $p_1\, p_2 \dots p_{m-1}\, p_m\, p_{m+1}\, p_{m+2}\, p_{m+3}$ his ~~s~~ sequence is $p_1\, p_2 \dots p_{m-1}$, ~~$p_m p_{m+1}$, $p_{m+2} p_{m+3}$, $p_{m+4} p_{m+5}$, $p_{m+6} p_{m+7}$~~. The number of integers of my sequence which are not divisible by p_r, $1 \le r \le m-1$ and which a[re] $< p_{m+9}^2$ are $p_m, p_{m+1}, p_{m+2}, p_{m+3}, p_m^2, p_{m+1}^2, p_{m+2}^2, p_{m+3}^2$ and $8+7+6+5$ more multiple[s] ($p_m p_{m+i}$, $1 \le i \le 8$, $p_{m+1} p_{m+i}$, $2 \le i \le 8$ etc) his ~~s~~ sequence contains 36 new numbe[rs] mine only 34

i.e. $\binom{9}{2}$

Surely for every k there are integers n for which

$$p_m\, p_{m+k+1} > p_{m+k}^2$$

but I think this may be hard to prove

$p_{m+i}\, p_{m+j}$, $0 \le i < j \le 8$

By the way one of our old problems stated: Let $a_1 < a_2 < \dots < a_k = n$ be such that no two a-s are relatively prime. We thought that max k is either $\frac{n}{p}$ wh[ere] p is the smallest prime factor of n or the numbers $2t$, $t \le \frac{n}{2}$, $(t, n) \ne 1$ give t[he] maximum, he disproved this too, but here too our conjecture can be modi[fied]

Kind regards to Fan + you, au revoir

E. P.

THE UNIVERSITY OF CALGARY
Faculty of SCIENCE
Department of MATHEMATICS & STATISTICS
2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4
Telephone (403) 284-5202

1982 IV 18

Dear Ron,
I enclose a cheque of 500 dollars.
Let $2^{\aleph_0} = \aleph_1$. Denote by $d(S_n)$ the set of distances between two points of S_n & I asked: can one divide r-dimensional space into $\aleph_0$ sets $\bigcup_{n=1}^{\infty} S_n$ so that all the distances in S_n are unique? i.e. ~~for~~ any four points of S_n determine 6 distinct distances.
Kakutani + I proved this 40 years ago for $r=1$ Davies proved it for $r=2$ and recently Kunen proved it for general r (it is not true in Hilbert space). Can one have that all the non-zero areas of the triangles in S_n should have distinct areas? Can you divide E_r into sets S_n, $n=1,2,\ldots$ so that no triangle in S_n should have area 1?

or no S_n should have all triangles of all the areas.*
If I remember right you proved that if the number of sets S_n is finite then one of the sets contain all the areas.

* In all these problems a modification is of course needed: Either you only ask that one S_n contains all the triangles of area $< \varepsilon$ or you ask for all areas but assume that all the sets are unbounded

A math in Kalamazoo asked: How many integers $1 \leq a_1 < \dots < a_k < \frac{n}{2}$ can one give so that $a_\ell \neq a_i + a_j$ and $a_u + a_v + a_w \neq n$.
If n is ~~odd~~ even you can take the odd numbers $< \frac{n}{2}$ but if n is odd I can not get more than $\frac{n}{6} + O(1)$ such integers.

I really can not sign the agreement because of the extra sheet - maybe I signed it once but then I did not notice it. I enclose another cheque which I just received.

Kind regards to all, au revoir

E. P.

Please send all mail until about May 13 to c/o Prof. G. Szekeres 94 Warragal road WARRAGAL TURRA-MURRA (near Sydney) N S W Australia

JAN [illegible]

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32000 Haifa, Israel

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חיפה 32000

Dear Ron,

Has anybody proved that the integers of the form $p^\alpha q^\beta$ form a complete set i.e every $n > n_0(p,q)$ is the sum of distinct numbers of the form $p^\alpha q^\beta$ also it would perhaps be of interest to try to estimate the size of $n_0(p,q)$

Please send 25 dollars to the enclosed adress. Hope you all are well

au revoir

E.P.

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