(Proper) coloring of a graph $=$ a way of coloring the vertices so that no two adjacent vertices have the same color.
$k$-coloring $=$ a coloring that uses exactly $k$ colors.
(If a $k$-coloring of $G$ exists, say that $G$ is $\boldsymbol{k}$-colorable.)
Chromatic number $\chi(G)=$ smallest $k$ so that $G$ is $k$-colorable.

