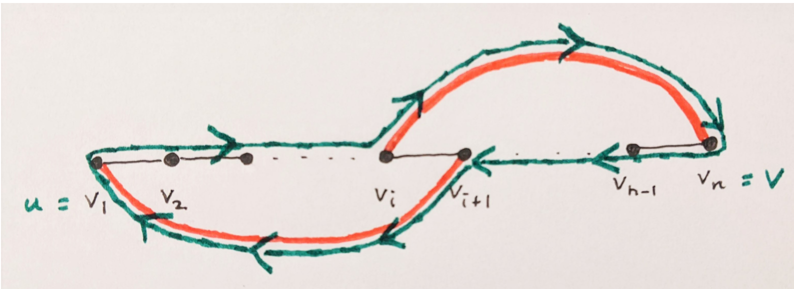
Dirac's Theorem proof recap: Want to show every graph with min degree $\geq n/2$ has a Hamiltonian cycle. **Strategy:**

(1) If we assume the theorem is false (for contradiction) and choose a maximal counterexample G, then **G has a Hamiltonian path** *(can you remember why?)*.

(2) To turn the Hamiltonian path into a Hamiltonian cycle, just need to find a carefully placed edge $\{v_i, v_{i+1}\}$ along the Hamiltonian path:



Formally, if $\{v_i, v_{i+1}\}$ is in **both** of these sets, we're done:

- $A = \{\{v_i, \mathbf{v_{i+1}}\} : \mathbf{v_{i+1}}$ is a neighbor of $u\}$
- $B = \{\{\mathbf{v_i}, v_{i+1}\} : \mathbf{v_i}$ is a neighbor of $v\}$

(3) Since u and v have high degree (both $\geq n/2$), A and B are large and must overlap somewhere:

- $|A| = \deg(u) \geq n/2$
- $|B| = \deg(v) \geq n/2$
- Only $n - 1$ edges along the Hamiltonian path to choose from: A, B are subsets of $\{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}\}$.