Let X be a Bernoulli random variable: $X = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$

(a) Compute the moments: $\mathbb{E}(X), \mathbb{E}(X^2), \mathbb{E}(X^3), \ldots$

(b) Compute the derivatives $M'_X(0), M''_X(0), M'''_X(0), \ldots$
(Remember: last time, we found that $M_X(t) = p \cdot e^t + (1-p)$)

Context: Last time, we argued that for a random variable X,

$$\begin{aligned} M_X(0) &= \mathbb{E}(X^0) \\ M'_X(0) &= \mathbb{E}(X^1) \\ M''_X(0) &= \mathbb{E}(X^2) \\ &\vdots \end{aligned}$$

This means that the Taylor series expansion of $M_t(X)$ is:

$$\sum_{k=0}^{\infty} \frac{M_X^{(k)}(0)}{k!} t^k = \sum_{k=0}^{\infty} \frac{\mathbb{E}(X^k)}{k!} t^k.$$

Equivalently, this series is the **exponential generating function** of the sequence of moments $\mathbb{E}(X^0), \mathbb{E}(X^1), \mathbb{E}(X^2), \ldots$