Math 180A Homework 0

Winter 2023

Due date: **11:59pm** (Pacific Time) on **Fri. Jan 13** (via Gradescope)

Welcome to your first homework assignment of Math 180A! This assignment is intended to give you practice with notation and concepts from calculus, set theory, combinatorics, and algebra that we will use at various points throughout the course, as well as practice submitting assignments on Gradescope.

If any of the techniques or notation are unfamiliar to you, or if you would like additional practice, there are links to review resources provided for each problem below. You are also welcome to come discuss these problems in office hours! Although this assignment will be graded for completion rather than correctness, you are encouraged to do it carefully, and make sure you really understand how to do each problem. It will be easier to do this now before things get busy later in the quarter!

Submission:

- To submit your completed work, log in to Gradescope with your `@ucsd.edu` email (either directly or through Canvas).
- Here is a guide to submitting an online assignment in Gradescope: `https://bit.ly/3WpTgfX`
- This assignment contains auto-graded questions (Section 1) for which you will input your answers directly into the Gradescope interface, and problems for which you will upload your full written solutions (Section 2). This is the format we will use for homework throughout the quarter. For Section 2, write each solution on a page by itself, and include only that solution in the file for the corresponding problem.
- Don't forget to double-check that your work was successfully submitted!
- **It will take some time to upload your work, so start early!** If you submit work early, you can always update your submission before the deadline.

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (Multiple choice)**.** Which of the following represents $\sum_{k=1}^{n} k(k+1)$?

(a) $1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1)$.

(b) $1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1)$.

(c) $(1 + 2 + \cdots + n)(2 + 3 + \cdots + (n+1))$.

Note: for a review of summation notation, you can look at Summation notation, from Paul's Online Notes.

Problem 2 (Select all that apply)**.** Consider the following table of values.

x_1^2	x_1x_2	x_1x_3	x_1x_4
x_2x_1	x_2^2	x_2x_3	x_2x_4
x_3x_1	x_3x_2	x_3^2	x_3x_4
x_4x_1	x_4x_2	x_4x_3	x_4^2

Which of the following are equivalent to the sum of the sixteen entries in the table? Choose all that apply. (*Recall:* the notations $\sum_{i=1}^{4} x_i$ and $\sum_{1 \le i \le 4} x_i$ mean $x_1 + x_2 + x_3 + x_4$.)

(a) $\displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} x_i x_j$

(b) $\displaystyle 2 \cdot \sum_{1 \le i \le j \le 4} x_i x_j$

(c) $\displaystyle\sum_{i=1}^{4} x_i^2 + 2 \cdot \sum_{1 \le i < j \le 4} x_i x_j$

(d) $\displaystyle\sum_{i=1}^{4} x_i x_j$

(e) $\displaystyle\left(\sum_{i=1}^{4} x_i\right)^2$

(f) $\displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} ij$

Note: if you would like some review on double summation, there is a very nice explanation from MIT OpenCourseWare. This video is about infinite rather than finite sums, but it contains very helpful graphics!

Problem 3 (Multiple choice)**.** Which of the following is a power series representation of e^{2x}?

(a) $2\sum_{k=0}^{\infty} \frac{1}{k!} x^k$.

(b) $\sum_{k=0}^{\infty} \frac{1}{k!} (2x)^k$.

(c) $\sum_{k=0}^{\infty} \frac{1}{(2k)!} x^k$.

Note: a power series representation of e^x is provided after Problem 7 below, together with a reference for review on power series.

Section 2 (upload PDF files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope. You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 4. Consider the region D of the coordinate plane defined by the following inequalities.

$$0 \leq x \leq 1 \qquad 2 \leq y \leq 4 \qquad \frac{x}{y} \geq \frac{1}{4}$$

(a) Draw a graph of the region D.

(b) Set up the limits of integration and evaluate the following integral over the region D.

$$\iint_D 6xy^2 \, dx \, dy$$

Note: For review on how to set up limits of integration for double integrals, or if you would like additional practice problems, you can look at Double integrals over a general region, from Paul's Online Notes.

Problem 5. Let A, B, and C be subsets of the set Ω. Various other sets are described below in words. Use unions, intersections, and complements to express these in terms of A, B, and C. Drawing Venn diagrams might help.

(a) The set of elements that are in each of the three sets.

(b) The set of elements that are in A but neither in B nor in C.

(c) The set of elements that are in at least one of the sets A or B.

(d) The set of elements that are in both A and B but not in C.

Note: For a review of set theory notation, you can look at Section 2.2 of *A First Course in Probability* by Ross, or Basic Set Operations, from Khan Academy. Also note that different authors may use $A^c, \overline{A}$, or A' to mean "the complement of A." In this course, we will use A^c, but all three are valid choices.

Problem 6.

(a) How many different binary strings of length 10 are there? *(A binary string is just a sequence of 0s and 1s; two examples of binary strings of length 10 are 1110000000 and 0000000111)*

(b) How many binary strings of length 10 have exactly five 1s?

Note: For a review of counting, you can look at Chapter 2 of Janko Gravner's notes, Sections 1.2-1.4 of *A First Course in Probability* by Ross, or Counting, permutations, and combinations from Khan Academy. We will cover some of this in class, but it will be pretty quick, so if it's your first time seeing it, I recommend spending some time studying counting outside of class (and as always, please come to office hours with questions!).

Problem 7. Give a closed form expression for each of the following power series.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{2k}}{4^k}$, where $x \in (-2, 2)$[1]

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{k+1}}{k!}$, where $x \in (-\infty, \infty)$

You may use the following series expansions from calculus without proof. (These are respectively the geometric series and the Taylor series of e^x centered at 0; they are the two infinite series we will see most often in this course.)

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}, \quad \text{when } x \in (-1, 1)$$

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x, \quad \text{when } x \in (-\infty, \infty)$$

Note: For a review of how to relate power series to functions, or if you would like additional practice problems, you can see Power Series and Functions, from Paul's Online Notes. In this tutorial, he works in the opposite order from what we are doing here (namely, he starts with a function in closed form and then represents it as a power series), but all the steps of the computations are the same.

[1] The symbol "$\in$" means "is in" or "is an element of."